B. QUANTITATIVE APTITUDE

LESSONS

SYLLABUS

PART 4 : QUANTITATIVE APTITUDE

Objective:

- To test basic understanding of Quantitative Aptitude.

Total Marks – 20

S. No.	*Topic*	*Sub Topic*
6	Number Systems	• Computation of Whole Number • Decimal and Fractions • Relationship between numbers
7	Fundamental arithmetical operations	• Percentages • Ratio and Proportion • Square roots • Averages • Interest (Simple and Compound) • Profit and Loss

CONTENTS

LESSON 6
NUMBER SYSTEMS

LESSON 7
FUNDAMENTAL ARITHMETICAL OPERATIONS

LESSON 6

NUMBER SYSTEMS

A number system is an arrangement of expressing the numbers in written form. Digits and Symbols in a consistent manner are used in number system. All the numbers are represented in the arithmetic and algebraic structure. The number system inter alia facilitates addition, subtraction, multiplication and division.

TYPES OF NUMBERS

The various types of numbers including the following:

1. Natural Numbers,
2. Whole Numbers,
3. Integers,
4. Rational Numbers,
5. Irrational Numbers,
6. Real Numbers and etc.

Let us discuss them in detail.

Natural Numbers

Natural numbers (N) are positive numbers i.e. 1, 2, 3 ….. and so one and so forth. Hence counting numbers in natural process like 1, 2, 3, ... constitute the system of natural numbers. These are the numbers which we use in our day-to-day life.

- It has to be noted there is no greatest natural number. For example, if 1 is added to any natural number, we get the next higher natural number, called its successor.
- Four-fundamental operations on natural numbers again generate natural number.

For, example, 4 + 2 = 6, again a natural number;

6 + 21 = 27, again a natural number;

22 – 6 = 16, again a natural number, but 2 – 6 is not defined in natural numbers.

Similarly, 4 × 3 = 12, again a natural number 12 × 3 = 36, again a natural number.

Four-fundamental operations of Natural Numbers are:

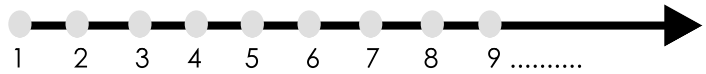
- Addition (Finding the Sum; '+')
- Subtraction (Finding the difference; '-')
- Multiplication (Finding the product; '×')
- Division (Finding the quotient; '÷')

12 divided by 6 (12/6) =2 is a natural number but 6 divided by 4 (6/4) is not defined in natural numbers.

Basis the above brief discussion on natural numbers, we can state following:

- Addition and multiplication of natural numbers again yield a natural number; but

- subtraction and division of two natural numbers may or may not yield a natural number
- The natural numbers can be represented on a number line as shown below.



- Two natural numbers can be added and multiplied in any order and the result obtained is always same. This does not hold for subtraction and division of natural numbers.

[1]Whole Numbers

When a natural number is subtracted from itself, and one cannot say what is the left out number. To remove this difficulty, the natural numbers were extended by the number zero (0), to get what is called the system of whole numbers.

- Whole numbers do not include any fractions, negative numbers or decimals.
- Again, like before, there is no greatest whole number.
- The number 0 has the following properties:

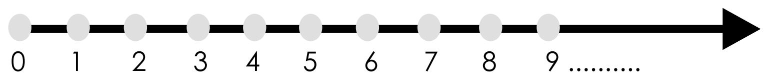
$$a + 0 = a = 0 + a$$

$a - 0 = a$ but $(0 - a)$ is not

defined in whole numbers a

$$\times 0 = 0 = 0 \times a$$

- Division by zero (0) is not defined.
- Four fundamental operations can be performed on whole numbers also as in the case of natural numbers (with restrictions for subtraction and division).
- Whole numbers can also be represented on the number line as follows:



Simple Natural Numbers and Whole Numbers Narration

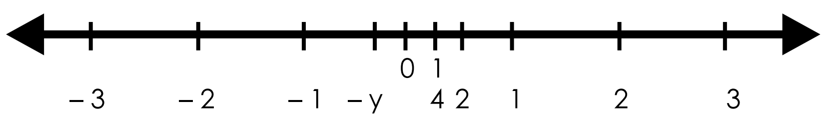
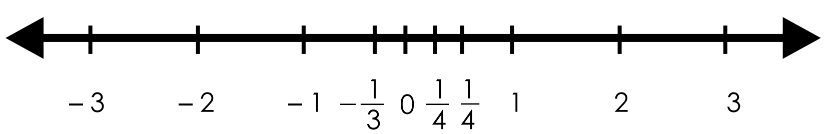


Figure 1: The number line

Assume one individual begins at 0 and continues along this number line in a positive direction. There are numbers everywhere that our eyes can see.



1. *https://www.nios.ac.in/media/documents/SecMathcour/Eng/Chapter-1.pdf*

Imagine that Sameer starts to move along the number line while also gathering some of the numbers for his bag. Sameer might start by only selecting natural numbers like 1, 2, 3, and so forth. Sameer is aware that this list is endless. Sameer's bag now holds an endless amount of natural numbers as a result of this accumulation. Students should be aware that we use the letter N to designate this collection. .

Pick up zero and place it in the bag, if Sameer turns around and walks all the way back now. You now possess the group of whole numbers, which is represented by the letter W..

Integers

It is discovered that subtracting one number from another isn't always possible when working with natural numbers and whole numbers. .

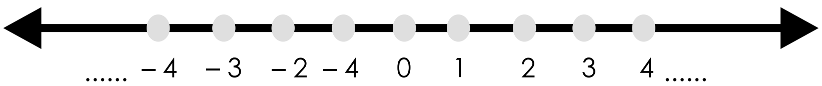
For instance, in the system of natural numbers and whole numbers, the numbers (5 - 7), (6 - 14), (18 - 38), and so on are all impossible. As a result, it required an additional extension of numbers that permit such subtractions. As a result, it is necessary to multiply whole numbers by negative numbers like -1 (also known as negative 1), - 2 (also known as negative 2), and so on. 5 + (–5) = 0, 10 + (–10) = 0, 15 + (–15) = 0..., 99 + (– 99) = 0, ...

As a result, we have expanded the whole number system to include another set of numbers known as integers. Therefore, the integers are: ..., – 7, – 6, – 5, – 4, – 3, – 2, – 1, 0, 1, 2, 3, 4, 5, 6, 7, ...

- It should be noted that although integers include negative numbers, they are equivalent to whole numbers in every way. Z represents them. .
- Examples: -3, -2, -1, 0, 1, 2

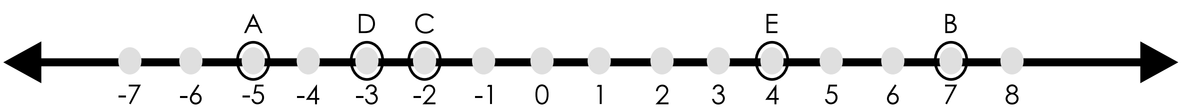
Natural Numbers, Whole Numbers and Integers: Representation on Number Line with Examples[2]

As extended number line is used for representing whole numbers to the left of zero and mark points – 1, – 2, – 3, – 4, ... such that 1 and – 1, 2 and – 2, 3 and – 3 are equal distant from zero and are in opposite directions of zero. Thus, we have the integer number line as follows:



Integers can be easily represented on the number line. For example, let us represent

– 5, 7, – 2, – 3, 4 on the number line. In the below mentioned figure, the points A, B, C, D and E respectively represent – 5, 7, – 2, – 3 and 4.



It is noted here that if an integer a > b, then 'a' will always be to the right of 'b', otherwise vice-versa.

For example, in the above figure 7 > 4, therefore B lies to the right of E. Similarly,

– 2 > – 5, therefore C (– 2) lies to the right of A (–5).

- On the other hand, since 4 < 7, 4 is located to the left of 7, which is depicted in the image as E is to the left of B. Hence, for finding the greater (or smaller) of the two integers a and b, following rule shall be observed:

2. *Source: Chapter 1 – Number Systems, Module – 1, Algebra, National Institute of Open Studies. https://www.nios.ac.in/media/documents/SecMathcour/Eng/Chapter-1.pdf 2*

i) a > b, if a is to the right of b

ii) a < b, if a is to the left of b

Example 1: Classify natural numbers, whole numbers and integers among the following: -

15, 22, – 6, 7, – 13, 0, 12, – 12, 13, – 31

Solution:

Natural numbers are: 7, 12, 13, 15 and 22

whole numbers are: 0, 7, 12, 13, 15 and 22

Integers are: – 31, – 13, – 12, – 6, 0, 7, 12, 13, 15 and 22

Note: From the above examples, we can say that

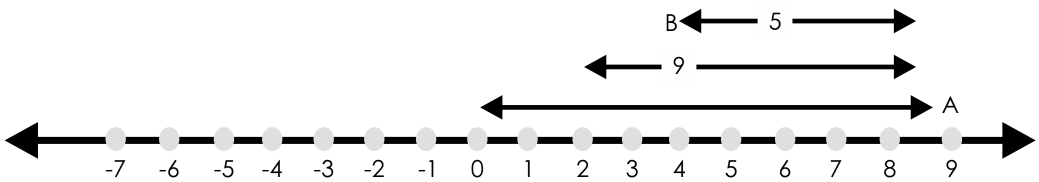
all natural numbers are whole numbers and integers also but the vice-versa is not true

all whole numbers are integers also

Example 2: Simplify the following and indicate whether or not the outcome is an integer. 12 × 4, 7/3, 18/3, 36/7, 14 × 2, 18/36, 13 × (–3)

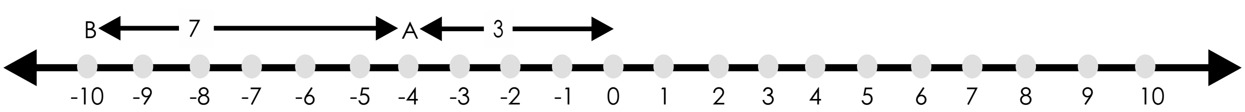
- **Solution:** 12 × 4 = 48 - it is an integer
- 7/3 - It is not an integer
- 18/3 = 6 - It is an integer
- 36/7 - It is not an integer
- 14 × 2 = 28 - It is an integer
- 18/36 - It is not an integer
- 13 × (–3) = – 39 - It is an integer

Example 3: Using number line, add the following integers: (i) 9 + (– 5) (ii) (– 3) + (– 7)



A represents 9 on the number line. Going 5 units to the left of A, we reach the point B, which represents 4.

Hence 9 + (–5) = 4



Moving three units to the left of zero while starting at zero, we arrive at point A, which stands for - 3. Going 7 units to the left of point A, we arrive at point B, which stands for -10. Hence (–3) + (–7) = – 10

[3]Rational Numbers

A number 'r' is called a rational number if it can be written in the form, where p and q are integers and q ≠ 0.

Consider the situation, when an integer a is divided by another non-zero integer b. The following cases arise:

- **When 'a' is a multiple of 'b'**
- Suppose a = mb, where m is a natural number or integer, then a/*b* = m
- **When a is not a multiple of b**
- In this case a/*b* is not an integer, and hence is a new type of number. Such a number is called rational number

Thus, a number which can be put in the form p/*q*, where p and q are integers and q/p, is called a rational number.

Note: Rational Numbers are of two types:

- **Positive Rational Numbers**

  A rational number p/q is said to be a positive rational number is p and q both are either positive integers or negative integers.

  Thus 3/4; 5/6; -3/-2; -8/-6, -12/57 are all positive rational numbers.

- **Negative Rational Number**

  If the integers p and q are of different signs, then p/q is said to be negative rational number. For example, -1/2; 6/-5; -12/4 and 16/-3 are all negative rational numbers.

Concluding Remark: (i) Every natural number is a rational number but the vice-versa is not always true.

(ii) Every whole number and integer are a rational number but vice-versa is not always true.

Irrational Numbers

From the previous discussion, we understand that there may be numbers on the number line that are not rational numbers. Hence, any number that cannot be expressed in the form of p/q, where p and q are integers and q ≠ 0, is an irrational number.

Examples: √2, 1.010024563… , e, π

We can also say that decimals numbers in any number line represent irrational numbers. Thus, *a decimal expansion which is neither terminating nor is repeating represents an irrational number.*

3. Source : NCERT Class 9, Chapter 1

Real Number

Any number which can be represented on the number line is a Real Number(R). It includes both rational and irrational numbers. Every point on the number line represents a unique real number.

TOPIC	DESCRIPTION
Natural Numbers	All counting numbers starting from 1,2,3,4,5……till infinity. The sum and multiplication product of two natural numbers is always a natural number; however, this doesn't stand are concerning subtraction and division.
Whole numbers	All counting numbers, including 0 (zero). These are also commonly called positive/non-negative integers. Like (0,1,2,3,4,5….)
Integers	The set of real numbers that consist of all-natural numbers, zero, and their additive inverses. (.....-5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5……)
Rational Numbers	All numbers that can be expressed as a ratio between two natural numbers in the form of fractions are called rational numbers. Like ½, □, ¾, etc. All terminable decimals are also rational numbers.
Irrational numbers	Numbers can not be written as fractions, decimals, or ratios. For e.g. Square roots, unending decimals (0.33333333...etc.), pie, etc.
Real numbers	These are numbers that include all of the above types of numbers. Rational, irrational, natural numbers, whole numbers, and so and so forth

[4]COMPUTATION OF WHOLE NUMBER

We have already discussed in detail that on a number line we use 1, 2, 3, 4,... when we begin to count. They come naturally when we start counting. Hence, mathematicians call the counting numbers as Natural numbers.

Predecessor and successor

Given any natural number, one can add 1 to that number and get the next number i.e. one

gets its successor. The successor of 16 is 16 + 1 = 17, that of 19 is 19 +1 = 20 and so on.

The number 16 comes before 17, hence it is said that the predecessor of 17 is 17–1=16, the predecessor of 20 is 20 – 1 = 19, and so on.

Whole Number

The number 3 has a predecessor and a successor. What about 2? The successor is 3 and the predecessor is 1. Does 1 have both a successor and a predecessor? The Answer is no. We have seen that the number 1 has no predecessor in natural numbers. To the collection of natural numbers, we add zero as the predecessor for 1.

Hence, The natural numbers along with zero form the collection of whole numbers.

4. *Source : NCERT*

Whole Numbers and Natural Numbers Comparison

It is evident from the definition above that all-natural Numbers are whole numbers, and that all whole numbers, with the exception of 0, are natural numbers. Set of Natural numbers:- {1,2,3,4……}

Facts to be known for computing Whole Number

a. All positive integers, including 0 are whole numbers.

b. Real numbers are all whole numbers. .

c. Whole numbers make up all of nature's numbers. All natural numbers except 0 begin with 1. .

d. Natural numbers are regarded whole numbers, although fractions, decimals, and negative numbers are not. .

e. The number zero is the only one that has no sign..

f. Fractions are not included in whole numbers because, as the name suggests, a whole number is neither a fraction nor a decimal. The full number is not a fraction as a result, hence it cannot be negative.

g. Another name for counting numbers is whole numbers.

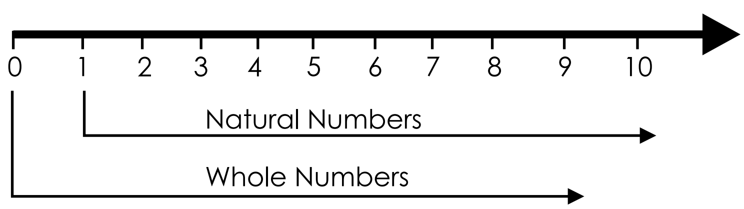
h. In mathematics, the numbers 0 through 1, 2, 3, and so forth stand in for the set of Whole Numbers.

i. The aforementioned facts demonstrate that all whole numbers and natural numbers are components of counting numbers. A whole number can also be obtained from the union of all positive counting integers plus zero.

j. A smallest whole number is 0 as it starts with zero (0).

k. The difference between the positive integer number line and negative integer number line is Zero.

Whole Number on Number line



Whole Number Calculation Techniques **Addition Property:** 0 does not change the final result. For Example- 2+0 = 2.

- **Closure Property:** Two Whole Numbers always produce a Whole Number as their product and their total. For example, 4 + 10 = 14 (A Whole Number), 4 * 10 = 40 (A Whole Number)
- **Associative Property:** The sum or product of the Whole Numbers remains the same regardless of how the numbers are organised. For example, 2 X 10 = 20 and 10 X 2 = 20 , 2 + 10 = 12 and 10 + 2 = 12, etc.
- **Multiplication Property:** The outcome of multiplying 1 by a whole number is that number itself. For example 7×1 = 7. If the whole number is multiplied by 0 then the result is 0. For example - 7×0 = 0.

- **Division Property:** When a whole number is divided by 0, the outcome is ambiguous. . For example- 7/0 = not defined.
- **Distributive Property:** This property is represented as P×(Q+R) = (P×Q) + (P×R). It is applicable for both addition and subtraction. For example - let P=11, Q=12, R=14, 11×(12+14) = (11×12)+(11×14) = 286.
- **Commutative Property:** P+Q = Q+P is a representation of this property. The property also holds for multiplication, but not for division or subtraction. For example - P=11, Q=12, 11+12 = 12+11 = 23.

Only one whole number, zero, is not a natural number. Until they are defined in terms of integers, fractions and negative numbers are not whole numbers.

Rounding of the Fractions

- Rounding off finds out the nearest whole number.
- For example: 7 is the closest whole number for 7.3.
- When the decimal number is less than .5, the whole number can be the number below the output.
- When the decimal point is .5 and above than, the whole number would be next whole number after rounding off.
- For examples 3.5 or 3.6 will become 4 after rounding off.

SAMPLE QUESTIONS ON WHOLE NUMBERS

Q1. A number in which one-fifth part is increased by 20 is equal to one-tenth part is increase by 30. Find the number.

a) 90

b) 100

c) 120

d) 150

Solution:- Let the number be x.

x/5 + 20= x/10+30

x/5 - x/10 = 10

x = 100

The number is 100. Option b) is correct.

Q2. The Product of two numbers is 150 and the sum of squares of numbers is 325. Find the sum of both numbers.

a) 24

b) 25

c) 29

d) 30

Solution:- let the two numbers be P and Q respectively.

$P \times Q = 150$ ……..(1)

$P^2+Q^2 = 325$ …..(2)

$(P+Q)^2 = P^2+Q^2+2PQ$

$(P+Q)^2 = 325+300$

$P+Q = 25$

Hence, option b) is the correct.

Q3. Which is the largest four-digit number divisible by 91?

a) 9919

b) 9900

c) 9909

d) None of these

Solution:- The Largest 4-digit number:- 9999

Largest four-digit number divisible by 91:- $91 \times 109 = 9919$

Q4. If the number 61xx4 is divisible by 6, then what will be the value of x?

a) 4

b) 5

c) 6

d) 7

Solution:- The number should have to be divisible by 2 & 3 both are divisible by 6. That is, if the last digit of the given number is even and the sum of its digits is a multiple of 3, then the given number is also a multiple of 6

Hence, option b) is correct.

Q5. Which of the following numbers is divisible by 13?

a) 1235

b) 1247

c) 1259

d) 1271

Solution:- Take the last digit of the number, multiply it by 4, and add the product to the rest of the number. If the answer is divisible by 13, then the number is also divisible by 13.

Hence, option a) is correct.

Q6. What is the value of 101, 104, 107…….134?

a) 1466

b) 1576

c) 1276

d) 1392

Solution:- All numbers are in Arithmetic Progression

Common difference d = 104-101 = 3

Final term an = a + (n-1)d

Where a is the first term, n is number of terms

134 = 101 + (n-1)3

33 = 3n-3

n = 12

Sum of numbers in A.P = Sn = n/2(a+l) where l = last term

Sn = 12/2 (101+131) = 1392

Hence, option d) is correct.

Q7. Write the smallest whole number.

Solution:

0 is the smallest whole number.

Q8. What is the predecessor of whole number 0?

Solution:

Whole number 0 has no predecessor.

[5]DECIMAL AND FRACTIONS

Key Points on Decimal and Fractions

- A fraction is a number representing a part of a whole.
- This whole may be a single object or a group of objects.
- A fraction whose numerator is less than the denominator is called a **proper fraction**, otherwise it is called an **improper fraction**.
- Numbers of the type $3\frac{5}{7}, 8\frac{4}{9}, 2\frac{1}{5}$ etc. are called **mixed fractions (numbers)**
- An improper fraction can be converted into a mixed fraction and vice versa.
- Fractions equivalent to a given fraction can be obtained by multiplying or dividing its numerator and denominator by a nonzero number.
- A fraction in which there is no common factor, except 1, in its numerator and denominator is called a fraction in the simplest or lowest form.

5. *Source: https://ncert.nic.in/pdf/publication/exemplarproblem/classVI/Mathematics/feep104.pdf*

- Fractions with same denominators are called like fractions and if the denominators are different, then they are called unlike fractions.
- Fractions can be compared by converting them into like fractions and then arranging them in ascending or descending order.
- Addition (or subtraction) of like fractions can be done by adding (or subtracting) their numerators.
- Addition (or subtraction) of unlike fractions can be done by converting them into like fractions.
- Fractions with denominators 10,100, etc. can be written in a form, using a decimal point, called decimal numbers or decimals.
- Place value of the place immediately after the decimal point (i.e., tenth place) is 1/10, that of next place (i.e., hundredths place) is 1/100 and so on.
- Fractions can be converted into decimals by writing them in the form with denominators 10,100, and so on. Similarly, decimals can be converted into fractions by removing their decimal points and writing 10,100, etc. in the denominators, depending upon the number of decimal places in the decimals. Decimal numbers can be compared using the idea of place value and then can be arranged in ascending or descending order.
- Decimals can be added (or subtracted) by writing them with equal number of decimal places.
- Many daily life problems can be solved by converting different units of measurements such as money, length, weight, etc. in the decimal form and then adding (or subtracting) them.

Convert Decimal to Fraction

To convert a Decimal to a Fraction, follow these steps:

- Step 1: Convert 0.50/ 1
- Step 2: Multiply each by 100

  50/100
- Step 3: Simplify 50/100

  =1/2

Examples on Fraction and Decimals (Showcasing Conversion and Steps of Conversion)

Example-1. Karan purchased 50 computers from a local computer market, only to discover that 10 of them were defective. Can you calculate the Fraction and Decimals of the defective computers in relation to the total computers purchased by Karan?

Answer: Out of 50 computers , we have 10 defective ones. As a result, the Percentage of defective computers is 10/50 . Now we must convert this Fraction to a Decimal. We must divide the Numerator 10 by the Denominator 50 to achieve this. As a result, by adding two Decimal places to the Fraction 10/50, it can be converted to a Decimal. 0.2 is the Decimal answer. As a result, the defective computers are 0.2 in Decimals.

Example-2. In an 100- office employees, 50 people chose burgers as a snack, while the other employees preferred mango juice. Calculate the Percentage of employees that choose a mango juice and give the result in Decimals.

Answer: There are 100 employees in an office, 50 employees who enjoy burgers, and 100 - 50 = 50 students who enjoy mango juice. mango juice are enjoyed by 50 percent of employees out of 100. This Fraction is equivalent to 2.5/5 on simplification. Let's convert this Fraction to a Decimal and then to a Percentage. To convert the Fraction to a Decimal, divide 2.5 by 5, and the result is 0.5. In order to convert 0.5to a Percentage, we must multiply it by 100, which is 0.5 x 100 percent = 50%. As a result, the Percentage of students who enjoy mango juice is 50%, and the Decimal equivalent is 0.5

Example-3. Write 1/2th in Decimals.

Answer: Let's look at how to express 1/2 in Decimals. To get a 100 in the Denominator, multiply the Numerator and Denominator with a 50. We also need to convert this Fraction to a Decimal with a Denominator of 100.

0.50 = 1/2 x 50/50 = 50/100

SAMPLE QUESTIONS ON FRACTIONS AND DECIMALS

Q 1: If 3/2 of a number is 9, find the number.

Solution:

Let x represents the required number.

Hence, 3/2 of x = 9

Therefore, multiply 9 with 2/3 = 6

Hence, the required number is 6.

Q 2: Multiply 1.73 and 2.7.

Solution:

173/100 × 270/100 = 4671/1000 = 4.671

Q3: Solve the following:

(a) 4 – 1/3

(b) 6 + 3/7

Solution:

(a) So, Subtract 4 with 1/3 i.e 0.33

4 – 1/3 = (12-1)/3 = 3 × 2/3

= 3.67

(b) In this case, Add 3/7 into 6

3/7 + 6 = 45/7 or = 0.4285+ 6

= 6.4285

Q4: The product of two numbers is 1.178. If one of them is 0.49, find the other number.

Solution:

Product of two numbers = 1.178

One number = 0.49

Other number = 1.178÷ 0.49

Hence, the required number = 2.40

Q.5: 1/4 of a number equals 4/5 ÷ 1/10. What is the number? (NCERT Exemplar)

Solution:

Let the number be x.

$$\therefore \frac{1}{8} of x = \frac{2}{5} \div \frac{1}{20}$$

$$\Rightarrow \frac{1}{8} \times x = \frac{2}{5} \div \frac{20^4}{1}$$

$$\Rightarrow \frac{1}{8} x = 2 \times 4 \Rightarrow \frac{1}{5} \times = 8$$

¼ of x = 4/5 ÷ 1/10

¼ of x = 4/5 * 10/1

¼ of x = 4 * 2 = 8

¼ of x = 8

X = 32

Hence, the required number = 32.

Q6: Simplify the following:

$$(i) \frac{2\frac{1}{2} + \frac{1}{5}}{2\frac{1}{2} \div \frac{1}{5}} \quad (ii) \frac{\frac{1}{4} + \frac{1}{5}}{1 - \frac{3}{8} \times \frac{3}{5}}$$

(NCERT Exemplar)

Solution:

$$(i) \frac{2\frac{1}{2} + \frac{1}{5}}{2\frac{1}{2} \div \frac{1}{5}} = \frac{\frac{5}{2} + \frac{1}{5}}{\frac{5}{2} \div \frac{1}{5}} = \frac{\frac{5x5 + 1x2}{10}}{\frac{5}{2} x \frac{5}{1}}$$

$$=\frac{\frac{25+2}{10}}{\frac{25}{2}}=\frac{27}{10_5}x\frac{2}{25}=\frac{27}{125}$$

$$(ii)\frac{\frac{1}{4}+\frac{1}{5}}{1-\frac{3}{8}x\frac{3}{5}}=\frac{\frac{5+4}{20}}{1-\frac{9}{40}}=\frac{\frac{9}{20}}{\frac{40-9}{40}}=\frac{\frac{9}{20}}{\frac{31}{40}}$$

$$=\frac{9}{20}x\frac{40^2}{31}x\frac{9x2}{31}=\frac{18}{31}$$

Solution:

Weight of the object on the Earth

$$= 5\frac{3}{5}\text{ kg} = \frac{28}{5}\text{ kg}$$

∴ Weight of the object on the Moon

$$=\frac{1}{6_3}x\frac{28^{14}}{5}\text{ kg} = \frac{14}{15}\text{ kg}$$

Hence, the required weight = $\frac{14}{15}$ kg.

DECIMAL FRACTIONS

Decimal fractions can be understood by considering normal fractions. A fraction has two parts numerator and denominator. It can be written as a/b.

Decimal fractions are those fraction in which denominator is 10, 100, 1000….. The numerator can be any number. These fractions are expressed in decimal numbers generally.

Examples of Decimal Fractions

- 9/100 can be expressed as 0.09.
- 180/100 can be expressed 1.80.
- 55/1000 is a decimal fraction written as 0.055.

Non-Examples of Decimal Fractions

Other fractions with non-ten numbers in the denominator are not decimal fractions. They are:

- 25/9
- 12/10125
- 91/125

Significance of Decimal Fractions

Decimal fractions inspire individuals to study about accurate quantities. Decimal Fractions assists us to recognize weights like 9.2 kg and distances like 5.55 km.

Conversion to Decimal Fractions

1. *Conversion from fractions to decimal fractions:*
   - Let us consider an example of a fraction, 9/2.
   - The first step would be to consider the number that gives 10 or a multiple of 10 when multiplied by the denominator. In this case, 5 multiplied by 2 gives 10.
   - Now multiply the numerator and denominator with the same number to get your decimal fraction. Here, 9 × 5/ 2 × 5 gives 45/10.
   - Thus, the decimal fraction of 9/2 is 45/10.
2. *Conversion from decimal numbers to decimal fractions:*

   Write the original decimal number in the numerator and denominator form by placing 1 in the denominator: 9.5/1.

   For every space that you move the decimal point, add a zero next to the 1 in the denominator: 95/10

   9.5/1

   95.0/10

   Once the number in the numerator is non-decimal, you have got your decimal fraction: 9.5 = 95/10.

Real-Life Application of Decimal Fractions

Decimal fractions are helpful for appreciating accurate quantities. They can be used to express percentages. For example, 85% can be written as 85/100.

Few situations where individual may come across decimal fractions:

- Coins (They are a fraction of Rupees)
- Weighing products

Solved Examples of Decimal Fractions

Example 1

Convert 5x1/2 into a decimal fraction.

= 5x 1/2

= 5/2

= 5 × 5 / 2 × 5

= 25 / 10

Example 2

Convert 9.9 into a decimal fraction.

= 9.9/1

= 99/10

RELATIONSHIP BETWEEN NUMBERS

In mathematics, a relation on a set may, or may not, hold between two given set members. For example, "is less than" is a relation on the set of natural numbers; it holds e.g. between 1 and 3 (denoted as 1<3) , and likewise between 3 and 4 (denoted as 3<4), but neither between 3 and 1 nor between 4 and 4. As another example, "is sister of" is a relation on the set of all people, it holds e.g. between Marie and Curie, and likewise vice versa. Set members may not be in relation "to a certain degree" - either they are in relation or they are not.

Practically in every day of our lives, we pair the members of two sets of numbers. For example, each hour of the day is paired with the local temperature reading by T.V. Station's weatherman, a teacher often pairs each set of score with the number of students receiving that score to see more clearly how well the class has understood the learning in the class.

Definition and Meaning - Relationship between Numbers

[6]Cartesian Product of Sets

Suppose A is a set of 2 colours and B is a set of 3 objects, i.e.,

A = {red, blue}and B = {b, c, s},

where *b, c* and *s* represent a particular bag, coat and shirt, respectively.

How many pairs of coloured objects can be made from these two sets?

Proceeding in a very orderly manner, we can see that there will be 6 distinct pairs as given below:

(red, *b*), (red, *c*), (red, *s*), (blue, *b*), (blue, *c*), (blue, *s*).

Thus, we get 6 distinct objects (*Fig 2.1*).

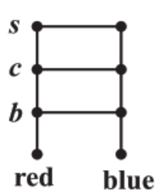


Fig. 2.1

Let us recall from our earlier classes that an ordered pair of elements taken from any two sets P and Q is a pair of elements written in small brackets and grouped together in a particular order, i.e., (p,q), $p \in P$ and $q \in Q$. This leads to the following definition:

Definition 1 Given two non-empty sets P and Q. The cartesian product P × Q is the set of all ordered pairs of elements from P and Q, i.e.,

6. *Source: https://ncert.nic.in/textbook/pdf/kemh102.pdf*

P × Q = { (*p*,*q*) : $p \in$ P, $q \in$ Q }

If either P or Q is the null set, then P × Q will also be empty set, i.e., P × Q = φ

From the illustration given above we note that

A × B = {(red, *b*), (red, *c*), (red, *s*), (blue, *b*), (blue, *c*), (blue, *s*)}.

Again, consider the two sets:

A = {DL, MP, KA}, where DL, MP, KA represent Delhi, Madhya Pradesh and Karnataka, respectively and B = {01, 02, 03}representing codes for the licence plates of vehicles issued by DL, MP and KA .

If the three states, Delhi, Madhya Pradesh and Karnataka were making codes for the licence plates of vehicles, with the restriction that the code begins with an element from set A, which are the pairs available from these sets and how many such pairs will there be (Fig 2.2)?

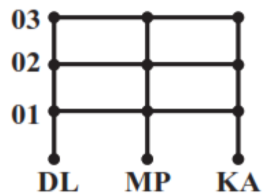


Fig 2.2

The available pairs are:(DL,01), (DL,02), (DL,03), (MP,01), (MP,02), (MP,03), (KA,01), (KA,02), (KA,03) and the product of set A and set B is given by

A × B = {(DL,01), (DL,02), (DL,03), (MP,01), (MP,02), (MP,03), (KA,01), (KA,02), (KA,03)}.

It can easily be seen that there will be 9 such pairs in the Cartesian product, since there are 3 elements in each of the sets A and B. This gives us 9 possible codes. Also note that the order in which these elements are paired is crucial. For example, the code (DL, 01) will not be the same as the code (01, DL).

As a final illustration, consider the two sets A= $\{a_1, a_2\}$ and

B = $\{b_1, b_2, b_3, b_4\}$ (Fig 2.3).

A × B = $\{(a_1, b_1), (a_1, b_2), (a_1, b_3), (a_1, b_4), (a_2, b_1), (a_2, b_2), (a_2, b_3), (a_2, b_4)\}$.

The 8 ordered pairs thus formed can represent the position of points in the plane if A and B are subsets of the set of real numbers and it is obvious that the point in the position (a_1, b_2) will be distinct from the point in the position (b_2, a_1).

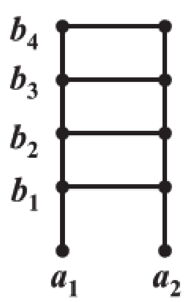


Fig 2.3

Remarks

(i) Two ordered pairs are equal, if and only if the corresponding first elements are equal and the second elements are also equal.

(ii) If there are p elements in A and q elements in B, then there will be pq elements in A × B, i.e., if $n(A) = p$ and $n(B) = q$, then $n(A \times B) = pq$.

(iii) If A and B are non-empty sets and either A or B is an infinite set, then so is A × B.

(iv) $A \times A \times A = \{(a, b, c) : a, b, c \in A\}$. Here (a, b, c) is called an ordered triplet.

Example 1 If $(x + 1, y - 2) = (3,1)$, find the values of x and y.

Solution Since the ordered pairs are equal, the corresponding elements are equal.

Therefore $x + 1 = 3$ and $y - 2 = 1$.

Solving we get $x = 2$ and $y = 3$.

Example 2 If $P = \{a, b, c\}$ and $Q = \{r\}$, form the sets P × Q and Q × P.

Are these two products equal?

Solution By the definition of the cartesian product,

$P \times Q = \{(a, r), (b, r), (c, r)\}$ and $Q \times P = \{(r, a), (r, b), (r, c)\}$

Since, by the definition of equality of ordered pairs, the pair (a, r) is not equal to the pair (r, a), we conclude that $P \times Q \neq Q \times P$.

However, the number of elements in each set will be the same.

Example 3 Let A = {1,2,3}, B = {3,4} and C = {4,5,6}. Find

(i) $A \times (B \cap C)$

(ii) $(A \times B) \cap (A \times C)$

(iii) $A \times (B \cup C)$

(iv) $(A \times B) \cup (A \times C)$

Solution

(i) By the definition of the intersection of two sets, $(B \cap C) = \{4\}$.

Therefore, $A \times (B \cap C) = \{(1,4), (2,4), (3,4)\}$.

(ii) Now $(A \times B) = \{(1,3), (1,4), (2,3), (2,4), (3,3), (3,4)\}$

and $(A \times C) = \{(1,4), (1,5), (1,6), (2,4), (2,5), (2,6), (3,4), (3,5), (3,6)\}$

Therefore, $(A \times B) \cap (A \times C) = \{(1, 4), (2, 4), (3, 4)\}$.

(iii) Since, $(B \cup C) = \{3, 4, 5, 6\}$, we have

$A \times (B \cup C) = \{(1,3), (1,4), (1,5), (1,6), (2,3), (2,4), (2,5), (2,6), (3,3), (3,4), (3,5), (3,6)\}$.

(iv) Using the sets A × B and A × C from part (ii) above, we obtain

$(A \times B) \cup (A \times C) = \{(1,3), (1,4), (1,5), (1,6), (2,3), (2,4), (2,5), (2,6), (3,3), (3,4), (3,5), (3,6)\}$.

Types of relations in mathematics

A relation R from a non-empty set A to a non empty set B is a subset of the Cartesian product A × B. The set of all first elements of the ordered pairs in a relation R from a set A to a set B is called the domain of the relation R. The set of all second elements in a relation R from a set A to a set B is called the range of the relation R. The whole set B is called the codomain of the relation R. Note that range is always a subset of codomain.

Types of Relations

A relation R in a set A is subset of A × A. Thus empty set φ and A × A are two extreme relations.

(i) A relation R in a set A is called empty relation, if no element of A is related to any element of A, i.e., $R = \varphi \subset A \times A$.

(ii) A relation R in a set A is called universal relation, if each element of A is related to every element of A, i.e., $R = A \times A$.

(iii) A relation R in A is said to be reflexive if aRa for all $a \in A$, R is symmetric if $aRb \Rightarrow bRa$, $\forall\ a, b \in A$ and it is said to be transitive if aRb and $bRc \Rightarrow aRc\ \forall\ a, b, c \in A$. Any relation which is reflexive, symmetric and transitive is called an equivalence relation.

Note: An important property of an equivalence relation is that it divides the set into pairwise disjoint subsets called equivalent classes whose collection is called a partition of the set. Note that the union of all equivalence classes gives the whole set.

Source: https://ncert.nic.in/pdf/publication/exemplarproblem/classXII/mathematics/leep201.pdf

Quick Examples of Relationship between Numbers

Order relations, including strict orders:

- Greater than
- Greater than or equal to
- Less than
- Less than or equal to
- Divides (evenly)
- Subset of

Equivalence relations:

- Equality
- Parallel with (for affine spaces)
- Is in bijection with
- Isomorphic

Tolerance relation, a reflexive and symmetric relation:

- Dependency relation, a finite tolerance relation
- Independency relation, the complement of some dependency relation
- Kinship relations

SAMPLE QUESTIONS ON RELATIONSHIP BETWEEN NUMBERS

Example – 1: If half of one-third of a number is 20, then half-tenth of that number will be:

1. 25
2. 50
3. 60
4. 80

Answer: 60

Explanation:

Let the number be x.

Then, 1/2 of 1/3 of x = 20

x = 20 x 2 x 2 = 120.

So, required number = 120 x 1/2 = 60.

Example – 2: The difference between a two-digit number and the number obtained by interchanging the positions of its digits is 54. What is the difference between the two digits of that number?

1. 2
2. 5
3. 6
4. None of these

Answer: 6

Explanation:

Let the ten's digit be x and unit's digit be y.

Then, (10x + y) - (10y + x) = 54

9(x - y) = 54

x - y = 6.

Example-3: The difference between a two-digit number and the number obtained by interchanging the digits is 54. What is the difference between the sum and the difference of the digits of the number if the ratio between the digits of the number is 4:1 ?

1. 4
2. 8
3. 16
4. None of these

Answer: Option 1 is correct

Example-4: A two-digit number is such that the product of the digits is 18. When 63 is added to the number, then the digits are reversed. The number is:

1. 19
2. 29
3. 39
4. 49

Answer is 2 i.e. 29

Example-5: The product of a two digit number is 14. When 18 is added to the number, then the digits interchange their places.

The number is:

(a) 68

(b) 59

(c) 95

(d) 86

The Answer is (a)

Example 6: If two fifth of one-third of a number is 50, then one-third of that number is:

(a) 376

(b) 3750

(c) 379

(d) 1250

Answer is (d) i.e. 1250

LIST OF FURTHER READING & REFERENCES

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- National Council of Educational Research and Training: https://ncert.nic.in
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LESSON 7

FUNDAMENTAL ARITHMETICAL OPERATIONS

INTRODUCTION

According to Britannica, Arithmetic (a term derived from the Greek word *arithmos*, "number") refers generally to the elementary aspects of the theory of numbers, arts of mensuration (measurement), and numerical computation (that is, the processes of addition, subtraction, multiplication, division, raising to powers, and extraction of roots). The purpose of these operations is to simplify mathematical expressions. Basic mathematical operations are plus, minus, multiply and divide. These operations are helpful in daily life. It covers many a things touching every aspect including calculating Income and Expenditure to preparation of Balance Sheets.

The BODMAS rule is one of the most important rule while doing arithmetical operations. B stands for Bracket (), O stands for Order, D stands for divide (÷), M stands for multiply (×), A stands for addition (+), S stands for subtract (-).

BASIC ARITHMETIC OPERATIONS

Mathematical Operations

The four arithmetic operations make up the fundamental mathematical operations. The inverse of addition is subtraction, and vice versa. This means that if two numbers are joined together to get a third number. Than we can find the of the number added by subtracting the other number from the total.

Example:

2 + 5 = 7

Now, if we subtract 2 from 7, we get;

7 – 2 = 5

Thus, we got the original number.

Similarly, multiplication and division are also inverse operations.

If 8 x 10 = 80

Then, 80/10 = 8

As a result, it is clear that these mathematical procedures are connected. These procedures are also the most straightforward kind of mathematic calculations, making them understandable to everyone.

Types of Fundamental Arithmetical Operation

Almost all forms of numbers, including integers, fractions, decimals, etc., can be subjected to arithmetic operations. Let's thoroughly comprehend each of the fundamental mathematical processes. The basic arithmetic operations in Mathematics are:

1. Addition (Finding the Sum; '+')
2. Subtraction (Finding the difference; '-')
3. Multiplication (Finding the product; '×')
4. Division (Finding the quotient; '÷')

Let us discuss all these four basic arithmetic operations with rules and examples in detail.

Addition Definition

A mathematical operation of adding items together is addition. The '+' sign indicates that something is being added. It entails adding two or more integers together to create a single number. The sequence is irrelevant when adding numbers. It indicates that addition is a commutative process. Any type of number, including real and complex numbers, fractions, and decimals, may be involved. Example: 9.12 + 1.88 = 11

The addition of more than two numbers, values or terms is also known as a summation of terms and can involve n number of values.

Addition Rules

The following are the addition rules for integers:

- A positive integer is the result of adding two positive numbers.
- A negative integer is created by adding two negative integers.
- When subtracting positive and negative integers, utilize the sign of the biggest integer value.

Subtraction Definition

The difference between two numbers is revealed by the subtraction procedure. A '-' symbol is used to indicate subtraction. It is addition done in the opposite direction. Subtraction is the act of combining a positive term with a negative term. The main purpose of this procedure is to determine how many remain after some items are removed. Example: 14 – 5

The term can also be re-written as 14 + (-5)

Adding terms we have, 7.

Subtraction Rules

The following are the subtraction rules for integers:

- Both the numbers are (+), the answer will also be positive
- Both the numbers are (-), the answer will also be negativeIf one number is positive and other number is negative, the answer will be in sign which is largest

Multiplication Definition

Repeated addition is another name for multiplication. It is indicated by a "x" or a "*". Additionally, it can combine with two or more other values to produce a single value. Multiplicand and multiplier are both involved in the multiplication process. The outcome of multiplying the multiplicand by the multiplier is referred to as the product. Example: 5 × 6 = 30

Here, "5" is the multiplier, "6" is the multiplicand, and the result "30" is called the product.

The product of two numbers says 'a' and 'b' results in a single value term 'ab,' where a and b are the factors of the final value obtained.

Multiplication Rules

The rules of multiplication are as follows:

- The product of two positive numbers is positive. In a multiplication if one number is positive and other number is negative, the answer will be negative.
- In a multiplication if both the numbers are negative, the answer will be positive.

Division Definition

The inverse of multiplication is division, which is typically represented by the symbol " ÷". It consists of the words dividend and divisor, where the value of the term is determined by dividing the dividend by the divisor. The result is larger than 1 if the dividend is greater than the divisor; otherwise, the result would be less than 1. Example: 8 ÷ 4 = 2

Here, "8" is the dividend, "4" is the divisor, and the result "2" is called the quotient.

Division Rules

The following are the division rules for integers:

A positive integer is obtained by dividing two positive integers. A positive integer is obtained by dividing two negative integers. The negative integer is produced when you divide two integers with different signs.

Basic Arithmetic Properties

The basic arithmetic properties for real numbers are:

1. Commutative property
2. Associative property
3. Distributive property

Commutative Property

This property is applicable only for two arithmetic operations, i.e., addition and multiplication.

Suppose A and B are two numbers, then, according to commutative property –

A+B = B+A	Example: 5 + 2 = 2 + 5
A x B = B x A	Example: 1 x 5 = 5 x 1

Thus, the order of numbers in addition and multiplication does not change the result.

Associative Property

Like commutative property, the associative property is also applicable to addition and multiplication.

A+(B+C) = (A+B)+C	Example: 1 + (5+3) = (1+5) +3
Ax(BxC) = (AxB)xC	Example: 1 x (5 x 3) = (1 x 5) x 3

Thus, if we change the grouping of numbers, the result does not change.

Distributive Property

According to the distributive property, if A, B and C are any three real numbers, then,

A x (B + C) = A x B + A x C

Example: 5 x (3 + 4) = (5 x 3) + (5 x 4)

5 x 7 = 15 + 20

35 = 35

Hence, proved.

SAMPLE QUESTIONS ON DMAS

1. Add 30 and 45 and then subtract 20 from the sum.

   Solution: On adding 30 and 45, we get;

   Sum = 30 + 45 = 75

   Now subtracting 20 from the sum, we get;

   75 – 20 = 55

2. *Solve:* 10 + 10 + 10 + 10 + 10.

   Solution: Given, 10 + 10 + 10 + 10 + 10

   It is clear that 10 is added to itself five times, thus, we can write;

   5 times of 10 = 5 x 10 = 50

   If we add them directly, the answer remains the same.

3. Find the value of (6 x 4) ÷ 12 + 72 ÷ 8 – 9.

   Solution: Given,

   (6 x 4) ÷ 12 + 72 ÷ 8 – 9

   ⇒ (24 ÷ 12) + (72 ÷ 8) – 9 [BODMAS rule[1]]

   ⇒ 2 + 9 – 9

   ⇒ 11 – 9

   ⇒ 2

4. *Simplify*: 24 - 4 ÷ 2 x 3

   Solution: 24 - 4 ÷ 2 x 3

   [Here order is expressed in short as 'DMAS' where 'D' stands for division, 'M' for multiplication, 'A' for addition and, 'S' for subtraction]

   = 24 - 2 x 3 [Performing division - 4 ÷ 2 = -2]

   = 24 - 6 [Performing multiplication 2 x 3 = 6]

   = 18. [Performing subtraction 24 – 6 = 18]

   Answer: 18.

1. BODMAS stands for Bracket, Order, Division, Multiplication, Addition, and Subtraction. The BODMAS is used to explain the order of operation of a mathematical expression.

5. *Simplify*: 24 ÷ 4 x 3 + 2

   Solution: 24 ÷ 4 x 3 + 2

   [Here order is expressed in short as 'DMAS' where 'D' stands for division, 'M' for multiplication, 'A' for addition and, 'S' for subtraction]

   = 6 x 3 + 2 [Performing division 24 ÷ 4 = 6]

   = 18 + 2 [Performing multiplication 6 x 3 = 18]

   = 20. [Performing addition 18 + 2]

   Answer: 20

6. *Simplify*: (-20) + (-8) ÷ (-2) x 3

   Solution: (-20) + (-8) ÷ (-2) x 3

   = (-20) + 4 x 3 [Performing division (-8) ÷ (-2) = 8 ÷ 2 = 4]

   = (-20) + 12 [Performing multiplication 4 x 3 = 12]

   = - 8. [Performing subtraction -20 + 12 = -8]

   Answer: -8

7. *Simplify*: (-5) - (-48) ÷ (-16) + (-2) x 6

   Solution: (-5) - (-48) ÷ (- 16) + (-2) x 6

   = (-5) - 3 + (-2) x 6 [Performing division (-48) ÷ (- 16) = 48 ÷ 16 = 3]

   = (-5) - 3 + (-12) [Performing multiplication (-2) x 6 = -12]

   = -5 - 3 -12

   = -8 - 12. [Performing addition -5 - 3 = -8]

   = -20 [Performing addition -8 - 12 = -20]

   Answer: -20.

8. *Simplify*: 52 - (2 x 6) + 17

   Solution:

   52 - (2 x 6) + 17

   = 52 – 12 + 17

   = 52 + 17 – 12

   = 57

   Answer: 57

PERCENTAGE

As per Britannica Dictionary, Percentage, is a relative value indicating hundredth parts of any quantity. One percent (symbolized 1%) is a hundredth part; thus, 100 percent represents the entirety and 200 percent specifies twice the given quantity.

For example, 1 percent of 1,000 Books equals 1/100 of 1,000, or 10 Books; 20 percent of the quantity is 20/100 1,000, or 200. These relationships may be generalized as x = PT/100 where T is the total reference quantity chosen to indicate 100 percent, and x is the quantity equivalent to a given percentage P of T. Thus, in the example for 1 percent of 1,000 Books, T is 1,000, P is 1, and x is found to be 10.

Percentage Formula (To calculate percentage of a number)

To determine the percentage, we have to divide the value by the total value and then multiply the resultant by 100.

Percentage formula = (Value/Total value) × 100

Example: 3/5 × 100 = 0.6 × 100 = 60 per cent

Example: if only 10 of the 200 apples are bad, what percent is that?

As a fraction, 10/200 = 0.05

As a percentage it is:

10/200 x 100 = 5%

To calculate the percentage of a number, we need to use a different formula such as:

- P% of Number = X
- where X is the required percentage.
- If we remove the % sign, then we need to express the above formulas as;
- P/100 * Number = X

Example 1: Calculate 20% of 80.

Let 20% of 80 = X

20/100 * 80 = X

X = 16

Example 2: Calculate 25% of 100

25% = 25/100

And 25/100 × 100 = 25

So, 25% of 100 is 25

Example 3: 30% of 400 apples are bad. How many apples are bad?

30% = 30/100

And 30/100 × 400 = 30 × 400/100 = 120

= 120 apples

120 apples are bad.

Example 4: A Mobile is reduced 25% in price.

The old price was Rs. 120. Find the new price.

First, find 25% of Rs. 120:

25% = 25/100

And 25/100 × Rs. 120 = Rs. 30

So, we have to reduce Rs. 30 from old price.

Hence the new price of Mobile is Rs. 120- Rs. 30 = Rs. 90

Variables of Percentage

Every percentage problem has three possible unknowns or variables :

- Percentage
- Part
- Base

In order to solve any percentage problem, you must be able to identify these variables.

Look at the following examples. All three variables are known:

Example 1: 80% of 20 is 16

- 80 is the percentage.
- 20 is the base.
- 16 is the part.

Example 2: 50% of 200 is 100

- 50 is the percent.
- 200 is the base.
- 100 is the part.

Example 3: 60 is 50% of 120

- 60 is the part.
- 50 is the percent.
- 120 is the base.

Increase or Decrease Percent[2]

We often come across such information in our daily life as.

(i) 25% off on MRP

(ii) 10% hike in the price of Diesel.

2. Source: Chapter 8, Maths Textbook, National Council of Educational Research and Training. Available at https/ncert.nic.in .

The original number is subtracted from the new number, divided by the original number, than multiplied by 100 to get the % increase. **% increase = [(New number – Original number)/Original number] x 100;**

where, increase in number = New number – original number

A percentage decrease is calculated by subtracting a new number from the original number, dividing that new number by the original number, and multiplying that result by 100. **% decrease = [(Original number – New number)/Original number] x 100**

Where decrease in number = Original number – New number

So basically, if the answer is negative then there is a percentage decrease.

Let us see few examples of such instances:

Example 1: The price of a laptop was INR 40,000 last year. It has increased by 25%this year. What is the price now?

Solution:

Option A: Let us first find the increase in the price, which is 25% of INR 40,000, and then find the new price.

25% of Rs. 40000 = 25/100 x 40000 = Rs. 10,000

New price = Old price + Increase = Rs. 40000 + Rs. 10,000 = Rs. 50,000

Option B - unitary method.

25% increase means – Rs. 100 increased to Rs. 125.

So, Rs. 40,000 will increase to?

Increased price = Rs. 125/Rs. 100 x 40,000 = Rs. 50,000

Similarly***, a percentage decrease in price would imply finding the actual decrease followed by its subtraction the from original price.***

Suppose in order to increase its sale, the price of laptop was decreased by 10%.

Then let us find the price of scooter.

Price of scooter = Rs. 40,000

Reduction = 10% of Rs. 40,000= Rs. 10/100 x 40,000 = Rs. 4000

New price = Old price – Reduction

= Rs. 40,000 – Rs. 4000 = Rs. 36,000

Finding Discounts in Percentage[3]

Discount is a reduction given on the Marked Price (MP) of the article.

Usually, this is done to encourage sales of the product or to entice customers to buy it. Subtracting the sale price from the listed price yields the discount. So, Discount = Marked price – Sale price

Example 1: A toy marked at Rs. 1,000 is sold for Rs. 920. What is the discount and discount %?

Solution: Discount = Marked Price – Sale Price

= Rs. 1,000 – Rs. 920

= Rs. 80

Since discount is on marked price, we will have to use marked price as the base.

On marked price of Rs. 1,000, the discount is Rs. 80.

On MP of Rs. 100, how much will the discount be?

Discount = 80/1000 x 100% = 8%

You can also find discount when discount % is given.

Example 2: The market price of a Key Board is Rs. 300. A discount of 15% is announced on sales. What is the amount of discount on it and its sale price.

Solution: Marked price is Rs. 300

15% discount means that on Rs. 100 (MP), the discount is Rs. 15.

By unitary method, on Rs. 1 the discount will be Rs. 15/100

On Rs. 300 , discount = Rs. 15/100 x 300= Rs. 45

The sale price = (Rs. 300 – Rs. 45) or Rs. 255.

Estimation in percentages

Let's say that the total bill of the restaurant is Rs. 919.78 and the restaurant gives a discount of 15%. How would we estimate the amount to be paid?

(i) Round off the bill to the nearest tens of Rs. 919.78, i.e., to Rs. 920.00

(ii) Find 10% of this, i.e., Rs. 10/100 x 920 = Rs. 92 =

(iii) Take half of this, i.e., ½ x 92 = Rs. 46

(iv) Add the amounts in (ii) and (iii) and we get Rs. 138

One could therefore reduce your bill amount by Rs. 138 which will be Rs.782 approximately.

Percentage vis-à-vis Fraction and Decimals

Examples of percentages at par with fractions are:

- 10% is equal to 1/10 fraction

3. *Source: Chapter 8, Maths Textbook, National Council of Educational Research and Training. Available at https/ncert.nic.in.*

- 20% is equivalent to 1/5 fraction
- 25% is equivalent to ¼ fraction
- 50% is equivalent to ½ fraction
- 75% is equivalent to ¾ fraction
- 90% is equivalent to 9/10 fraction

There is no dimension to percentages. As a result, it is known as a dimensionless number. As in 0.47%, 0.90%, etc., percentages can also be expressed as decimals or fractions. The grades earned in any topic are calculated in terms of percentages in academics. Shilpa, for instance, scored 65% in her final exam. This percentage is derived based on Shilpa 's overall grade point average (GPA) across all disciplines.

SAMPLE QUESTIONS ON PERCENTAGE

Q.1: If 20% of 30% of a number is 8, then find the number.

Solution:

Let X be the required number.

Therefore, as per the given question,

(20/100) × (30/100) × X = 8

So, X = (8 × 100 × 100) / (20 × 30)

= 133.33

Q.2: Which number is 30% less than 75?

Solution:

Required number = 70% of 75

= (75 x 70)/100

= 52.5

Therefore, the number 52.5 is 30% less than 75.

Q.3: The sum of (12% of 28.6) and (5% of 1.75) is equal to what value?

Solution:

As per the given question,

Sum = (12% of 28.6) + (5% of 1.75)

= (28.6× 12)/100 + (1.75 × 5)/100

= 3.432 + 0.0875

= 3.5195

Q.4: A stationery seller had some registers. He sells 40% registers and still has 420 registers . Originally, he had how many registers ?

Solution:

Let he had N registers, originally.

Now, as per the given question, we have;

(100 – 40)% of N = 420

$\Rightarrow$ (60/100) × N = 420

$\Rightarrow$ N = (420 × 100/60) = 700

Q.5: Out of two numbers, 20% of the greater number is equal to 50% of the smaller. If the sum of the numbers is 140, then the greater number is?

Solution:

Let X be the greater number.

$\therefore$ Smaller number = 140 – X {given that the sum of two numbers is 140}

According to the question,

(20 × X)/100 = 50(140 – X)/100

$\Rightarrow$ 100

RATIO AND PROPORTION

The fractional numbers are the foundation of ratio and proportion. A ratio is a fractional number that is stated in the form a: b. The mathematical notions of ratio and proportion serve as the foundation for understanding many other mathematical ideas. We frequently use the concepts of ratio and proportion in daily life, such as when negotiating a financial agreement, comparing our heights and weights to those of others, adding ingredients while we prepare meals in the kitchen, etc.

Ratio and Proportion – Usage

We frequently find the concepts of ratio and proportion to be confusing. A ratio is the result of comparing two parameters side by side using the division operator. A proportion is the resemblance of two separate ratios in terms of value. A ratio can also be stated differently, for example, as x:y or x/y. It should be understood as x is to y. A proportion, on the other hand, is a mathematical formula that declares that two ratios are equal. An expression for a percentage is x: y:: p: q. It should be understood as x is to y as p is to q. In this case, the denominators y & q are not numerically comparable to 0.

Definition of Ratio

When two parameters are compared, a ratio is created by applying the division operator to the first and second values. The quotient x/y is typically referred to as the ratio between x and y when x and y are two parameters of the same type and with similar units, such as y is not equivalent to 0. The colon (:) symbol is used to denote ratios. It implies that the ratio x/y can be written as x: y and has no units. To put it another way, the ratio is the number used to represent one quantity as a fraction of the other item. Only if the two quantities in a ratio have the same unit can they be compared.

Definition of Proportion

A proportion is a mathematical phrase that indicates the two ratios are comparable to one another. The similarity between the two fractional numbers or ratios is, to put it simply, the proportion. The two ratios are intended to be directly proportionate to one another when the two sets of specified quantities are changed in a similar way. The symbol (::) represents proportions and aids in figuring out ambiguous numbers.

Types of Proportion

There are two types of proportions as follows.

1. Direct Proportion

The term "direct proportion" describes the direct correlation of the two numbers. When one number rises, the other rises as well, and vice versa. For instance, if a vehicle's speed is raised, its distance traveled will undoubtedly increase.

2. Inverse Proportion

The term "inverse proportion" describes how two numbers are related in such a way that when one number rises, the other number falls, and vice versa. As a result, the inverse ratio is written as a 1/b. For instance, if we drink more water from a bottle, there will be less water left in the bottle overall.

Ratio and Proportion Formula

The Ratio Formula is written as $x : y \Rightarrow x/y$ where

x = Antecedent or the first term

y = Consequent or the second term

> For example, Ratio 8 : 4 is also written as 8/4, where 8 is called the antecedent and 4 is called the consequent.

In order to write a proportion in mathematics for the two ratios, a:b and y:z then we express it as a:b :: y:z ⟶ a/b = y/z

- The two numbers namely b and y are called the **mean terms**.
- The two numbers namely a and z are called the **extreme terms**.
- In a : b = y : z, the numbers or parameters of a and b should be of the same type with similar units, while y and z may be the separate ratios of parameters of the same type with similar units. For example, 10 meter: 20 meter = 50 kg: 100 kg.
- In the concept of proportion, the product of the mean terms is equivalent to the product of the extreme terms. Hence, we get $b \times y = a \times z$.
- For example, In the proportion of two ratios of 5 : 10 :: 10 : 20, we apply the formula of The Product of Mean Terms = The Product of Extreme Terms

  We get, $10 \times 10 = 5 \times 20 = 100$
- The proportion formula can be written in the form of a/b = c/d or a : b : : c : d.

Difference between Ratio and Proportion

When it comes to mathematics, the concepts employed in ratio and proportion are similar, hence they are regarded as one subject. Students occasionally struggle to understand the concepts of ratio and proportion. For better comprehension, consider the following comparison of ratio and proportion.

Sr. No.	Ratio	Proportion
(i)	When comparing various quantities with the same units, it is used.	It is used to describe a relationship between two ratios, each of which may have a different set of units. It is used to describe a relationship between two ratios, each of which may have a different set of units.
(ii)	To express a ratio, two symbols are used: a colon (:), and a slash (/).	It is possible to express a proportion using the double colon (::) symbol.
(iii)	It is defined as an expression.	It is termed as an equation.

Key Notes on Ratio and Proportion

By employing the idea of ratio, any numbers or parameters with comparable units can be compared. Only when two ratios are the same we can say that they are in a proportional relationship. You may also use the cross-multiplication approach to determine whether two ratios are equal and how they stack up in terms of proportion. A ratio always produces equivalent outcomes when the individual numbers are multiplied and divided by like numbers.

SAMPLE QUESTIONS ON RATIO AND PROPORTION

Question 1: There are 63 students available in the 8th class. The number of students who want to study Sanskrit and the number of students who want to study Mathematics is expressed in the ratio 5:2. Calculate the number of students who want to study Sanskrit and those who want to study Mathematics.

Solution: Given that, the total number of students in the 8th class = 63

Let the number of students who want to study Sanskrit = 5x and

the number of students who want to study Mathematics = 2x

As per the question, we can say that $5x + 2x = 63 \Rightarrow 7x = 63 \Rightarrow x = 9$

After putting the value of x = 9,

we get the numbers of students who want to study Sanskrit = $5x = 5 \times 9 = 45$ and the number of students who want to study Mathematics = $2x = 2 \times 9 = 18$

Hence 45 students of class 8th want to study Sanskrit and 18 students want to study Mathematics .

Question 2: R and S started an Electronic shop and decided to divide the profit between them in a ratio of 7:5. The total profit from that shop is Rs. 12,000 by the end of the financial year 2022. What will be the individual profit share for both R and S?

Solution: The total profit earned from the shop is to be divided between R and S in a ratio of 7:5

Hence, we can calculate the individual profit of both persons by

R= 12,000 x (7/12) = 7000

S = 12,000 x (5/12) = 5000

Hence the individual profit for both person R and S will be 7000 and 5000

Question 3: If Abhishek travels a distance of 25 km in 5 hours. How much distance can he travel in the time of 8 hours?

Solution: Let us consider the traveling distance to be z in the time of 8 hours. With time, the traveling distance is also increased.

So, 5 : 8 = 25 : z

z = (25 x 8) / 5

= 40 km

Thus, Abhishek can travel a distance of 40 km in 8 hours.

Question 4: Calculate the numbers whose sum is 88 and they are written in the ratio of 4:4

Solution: Let us consider the numbers to be 4x and 4x, respectively.

As per the question, the sum of the considered two numbers is 88.

Now, 4x + 4x = 88

8x = 88

x = 11

Hence the two numbers will be

4x = 4 x 11 = 44

4x = 4 x 11 = 44

44 and 44 are the two numbers that satisfy the given statement of the question.

SQUARE ROOTS

When an integer is multiplied by itself, the result is known as a square root. The result of a number multiplying itself is referred to as the square number. The symbol for a square root is a √ sign.

Square Root Definition

Any square root when multiplied by the same number, the result is the original number. For a perfect square number, we obtain perfect square roots.

Example: 1^2 = 1, therefore square root of 1 becomes 1.

2^4 = 4, therefore square root of 4 becomes 2.

Similarly, 9^2 = 81, therefore the square root of 81 becomes 9.

It is also important to note that 9^2, gives 81, and − 9^2 also gives 81.

Methods to Find Square Root of Numbers

- To determine whether a number is a perfect square or an imperfect square, one must first determine the number's square root. A perfect square is defined as a number that can be expressed as the square of the number from the same number system.
- Imperfect squares are those numbers whose square roots contain fractions or decimals.

The prime factorization method can be used to factorize a number if it turns out to be a perfect square. There could be 25, 36, 4, 81, etc. perfect squares. The square root via long division method will be employed to determine the square root of an integer, albeit, if it is an imperfect square. Examples of imperfect squares include 2, 3, 5, 7 and others. Some of the key methods to find out the square root of a number are as follows:

1. Repeated Subtraction Method
2. Prime Factorization
3. Estimation Method
4. Long Division Method

Repeated Subtraction Method

One of the methods frequently used to determine the square root of a number is repeated subtraction. This approach involves repeatedly subtracting the perfect square number from subsequent odd integers, such as 3, 5, 7, 9, etc., until the result is zero. Starting with 1, the subtraction proceeds through 3, 5, 7, and so forth until 0 is reached. This approach counts how many times the value is deducted from one to get to zero. This count indicates the required square root of the given numbers.

36 – 1	35
35-3	32
32-5	27
27-7	20
20-9	11
11-11	0

The sum of the six subtraction operations is 0, as can be seen in the table above. Starting with 1, the subtraction continues until the odd number, 11, is reached. In total, 1, 3, 5, 7, 9 and 11 are deducted. This represents 6 occurrences. 6 is therefore the square root of 36.

Prime Factorization Method

The prime factorization method is a simple way to get a number's square root. By dividing the perfect square progressively, this approach divides it into its prime factors. The prime factor pairs are then paired. The square root of the perfect square is obtained by multiplying one element from each pair. Let us find the square root of 196.

The prime factorization of 196 = $2 \times 2 \times 7 \times 7$.

When we pair the prime factors and select one from each pair, we have $7 \times 2 = 14$. Hence, the square root of 196 is 14.

Estimation Method

An approximation method is the square root by estimation method. By making educated guesses about the values, this approach determines the square root of numbers. Taking 4 as an example, the square root is 2, while 9 is the square root, which is 3. Knowing that the square root of 5 will be between 2 and 3 is therefore simple. However, we will still have to check the value of √6 is nearer to 2 or 3.

> Let us try finding out the square of 2.4 and 2.9.
>
> The square of 2.4 = 5.76
>
> The square of 2.9 = 8.41
>
> Since the square of 2.4 is 5.76, which is approximately 6, we can say that the square root of 6 is approximately equal to 2.4.

Long Division Method

Finding the square root of numbers that are not perfect squares is challenging. However, using the long division method makes this simple to accomplish.

By calculating the square root of 225, let's examine the procedures involved in finding the square root by long division. Starting at the unit position, first add a bar over the digits in 225.Starting from the left-most part of the integer, divide. 1 is the integer in this case whose square is smaller than 2. The outcome will be as follows when it is divided by the quotient and doubled.

	16
	256
1	1
26	156
	0

Properties of Square Root

- Only a perfect square number can have a perfect square root.
- An even perfect square has an even square root.
- The square root of an odd perfect square will be odd.
- Because a perfect square cannot be negative, it is impossible to define the square root of a negative number.
- A square root can be found for any number that ends in the digit of the unit, such as 1, 4, 5, 6, or 9.
- It is impossible to obtain a perfect square root if the unit digit of an integer is 2, 3, 7, or 8.
- A number cannot have a square root if it has an odd number of zeros at the end. Only an even number of zeros allows for the calculation of a square root.

Square Root

Formula

To determine the square root of a number, use the square root formula. The square root formula is y= √ x to make things easier. It is important to note that y x y=x. Here x is the square of a number y.

For e.g., 2 = √4, where y = 2 and √x = 4, thus y x y= x, i.e. 2×2 = 4.

Square Root of a Negative Number

Understanding that negative numbers also have square roots is important. Negative square roots, on the other hand, are complex numbers rather than real numbers. This is so because any integer's square is a positive number. For instance, "-x"'s primary square root is "(-x)=ix." Here, "i" represents the square root of -1.

Let's examine another illustration. The square root of a perfect square integer like 16 is taken into account. Let's think about the square root of -16 now. The integer -16 has no true square root. √(-16)= √16 × √(-1) = 4i (as, √(-1)= i)

Here, "i" is represented as the square root of -1. Hence, 4i is the square root of the number 16.

SAMPLE QUESTIONS ON SQUARE ROOT

1. **Which of the following figures is a square in all its parts?**

   a) 111

   b) 225

   c) 142

   d) 156

   Answer: Option (B)

2. **A perfect square number can never have the digit ….. at the units place.**

   a) 1

   b) 4

   c) 8

   d) 9

   Answer: Option (C)

3. **Evaluate √ 6241**

   a) 72

   b) 75

   c) 78

   d) 79

   Answer: Option (D)

4. **Find the square root of 6724.**

   a) 79

   b) 76

   c) 82

   d) 87

   Answer: Option (C)

5. **Evaluate √ 1498176**

   a) 1216

   b) 1224

   c) 1215

   d) 1223

   Answer: Option (B)

AVERAGE

In plain English, an average is a single number chosen to represent a group of numbers. This average is typically the arithmetic mean, which is the total of the numbers divided by the number of numbers in the group. The average of the numbers 2, 3, 4, 7, and 9 (which add up to 25) is, for instance, 5. An average could be another statistic like the median or mode depending on the situation. In mathematics, the central value of a set of data is expressed as the average of a list of data. It is defined mathematically as the ratio of the total number of data points to the number of units in the list.

It is fairly simple to calculate the average of a set of numbers or values. After adding up all the numbers, divide the total by the number of values provided. Consequently, the following is the math average formula: Average = Sum of Values/ Number of values

Assume that we have provided n different values, such as x1, x2, x3,..., xn. The data will have the following average or mean: Average = (x1+x2+x3+…+xn)/n

Formula to Calculate Average

For a given set of variables, we can quickly calculate the average. Simply add up all the values, then divide the result by the total number of values. Average can be calculated using three simple steps. They are:

- Step 1: Sum of Numbers:

  Finding the sum of all the given numbers is the first step in calculating the average of a set of numbers.

- Step 2: Number of Observations:

  The next step is to determine how many numbers are there in the dataset.

- Step 3: Average Calculation:

  In order to arrive at the average, divide the total by the number of observations. Now, let us consider an example to calculate the average.

If there are a group of numbers say, 19, 25, 29, 21, 22. Then find the average of these values.

By average formula, we know,

Average = (Sum of values)/No. of values

= (19+25+29+21+22)/5

= 116/5

=23.20

Arithmetic Mean

The most typical kind of average is called the arithmetic mean. The arithmetic mean is the sum of the as divided by n where n is a number. If n numbers are supplied, each number denoted by ai (where i = 1, 2,..., n), then:

Where,

- n is the number of observations
- i represent the index of summation
- and ai = data value for the given index

Geometric Mean

- By determining the nth root of the product of n numbers, the geometric mean is a technique for determining the central tendency of a set of numbers. In contrast to the arithmetic mean, which is calculated by adding the observations and then dividing the total by the number of observations, it is fundamentally different. However, in the case of the geometric mean, we first calculate the product of all observations before calculating the nth root of the product, assuming that n is the number of observations. The formula is given by - Geometric Mean,
- x1, x2, x3,…, xn are the individual items up to n terms

Harmonic Mean

The reciprocal of the average of the reciprocals of the given data values is referred to as the harmonic mean. The formula to find the harmonic mean is given by:

Harmonic Mean, HM = n / [(1/x1) + (1/x2) + (1/x3) +…+(1/xn)]

Where x1, x2, x3,…, xn are the individual items up to n terms.

Average of Negative Numbers

The procedure or formula to calculate the average is the same if the list contains any negative integers. Let’s use an example to better grasp this. Example:

Find the average of 3, −7, 8, 12, −2.

Solution: The sum of these numbers

= 3 + (-7) + 8 + 12 + (-2)

= 3 – 7 + 8 + 12 – 2

= 14

Total Units = 5

Hence, average = 12/5 = 2.8

Solved Examples on Averages

- **Example 1**: Find the average of 7, 3, 8, 9, 8

  Solution:

  Add the numbers = 7, 3, 8, 9, 8 = 35

  Total Units = 5

  Hence, average = 35/5 =7

- **Example 2**: Find the average of 11,13, 19, 22, 10

  Solution:

  Add the numbers

  = 11+13+19+22+10 = 75

  Total units = 5

  Hence, average = 75/5 = 15

- **Example 3:**

  If the age of 9 boys in a team is 12, 13, 11, 12, 13, 12, 11, 12, 12. Then find the average age of boys in the team.

  Solution:

  Given, the age of boys are 12, 13, 11, 12, 13, 12, 11, 12, 12.

  Average = Sum of ages of all the students/Total number of students

  A = (12+13+11+12+13+12+11+12+12)/9

  A = 108/9

  A = 12

  Hence, the average age of boys in a team is 12 years.

- **Example 4:**

  If the heights of females in a group are 5.1, 5.2, 5.6, 5.4, 5.9, 5.8, 5.10, 5.5, 6, 5.3. Then find the average height.

  Solution:

  Given the height of females: 5.1, 5.2, 5.6, 5.4, 5.9, 5.8, 5.10, 5.5, 6, 5.3 Average = Sum of heights of males/total number of females

A = 5.1+5.2+5.6+5.4+5.9+5.8+5.10+5.5+6+5.3 10

A = 54.9/10

A = 5.49

INTEREST (SIMPLE AND COMPOUND)

The maxim that one should never borrow money for free should be kept in mind at all times. Since everyone has a need for money, borrowing money is expensive. Therefore, interest is the sum of money paid for using someone else's funds. You must pay interest when you borrow money from lenders. You receive interest when you lend money to borrowers.

In this section the aim is to learn - what is interest, types of interest, and how to calculate the interest amount.

Meaning and definition of Interest

Interest is the extra sum that a borrower pays to a lender in addition to repaying the amount borrowed. For instance, a borrower might take out a loan for Rs. 10,000 and agree to pay an additional Rs. 100 as interest. The sum of interest received or paid over a predetermined period is known as an interest rate. The interest rate, for instance, would be 10% if the prior borrower agreed to pay the debt in full within a year.

Types of Interest Rate

Interest can be simple or compounded. The calculation for both type of interests is different. Further, compound interest can be computed on a daily, weekly, monthly, biweekly, quarterly, or even annual basis.

How does Interest Work

There are various methods for calculating interest, some of which are more advantageous for lenders. The amount of interest you receive depends on the alternative investment possibilities you have open to you, whereas the amount of interest you pay depends on what you anticipate receiving in return. **When Borrowing** - When one borrows money, they are required to pay it back. One must pay back more than they originally borrowed plus interest in order to make up for the lender's risk in lending the money.

When Lending - If someone has extra money, they can lend it out or put it in a savings account, which allows the bank to lend it out or invest the money. One anticipates receiving interest in exchange for lending or depositing money.

Amount of interest you pay or earn depends on the following factors:

- The rate of interest.
- The amount of loan.
- How long does it take to repay loans?

If you use a basic interest formula to determine your interest amount, an interest charge of Rs. 500 will be applied to a loan of Rs. 5000 with an annual interest rate of 10%. Compound interest, which is used by the majority of banks and credit card companies instead of basic interest, causes interest payments to increase more quickly.

Formula to Calculate Interest

Interest can be calculated using two methods. These two methods are:

- Simple Interest
- Compound Interest

Simple Interest

Simple interest is a way of calculating interest on the amount borrowed or invested for the duration of the loan without taking into account any extra variables, such as prior interest (paid or charged) or any other financial considerations. On the initial principal sum, simple interest is paid; it is not compounded. A short-term loan, often one year or less, that is administered by financial companies or money invested for a comparable short-term length is typically subject to simple interest. The formula for calculating simple interest is

Simple Interest (SI) = P(Principal) × R(Rate of Interest) × T(Interest Period) / 100

Here, P stands for the principal sum, R for the rate of interest, and T for the period of interest.

The total amount due in the end is made up of the principal plus the simple interest, or P + SI. For example,

Q. An invested sum fetched a total interest of INR 10000 at the rate of 10% in one year. What was the original principal amount?

Solution: Let principal amount be P, SI be simple interest, R be the rate of interest, and T the time period.

Accordingly, SI = PRT/100

10000 = P × 10 × 1/100

P = 1000000 /10

P = 100000

Hence, the original principal amount is INR 100000.

Simple Interest Formula for Months

The above formula can be used to determine simple interest on an annual basis. Let's now look at the method for calculating interest over a period of months. If P is the initial investment, R is the annual interest rate, and n is the duration (in months), then the following formula can be written: Simple Interest for 9 months = (P × 9 × R)/ (12 ×100)

Compound Interest

In general, interest rates on investments are compound interest; interest is calculated on the amount due at the time of calculation rather than the original principal. Interest is said to compound in this manner. The investor benefits from compound interest since it allows them to earn extra money in addition to their original principle investment. The interest that is imposed on another interest is essentially what is meant by compound interest. If John take Rs. 4000 per year at a 10% interest rate, the interest for the first year will be equal to 10% of Rs. 4000, or Rs. 400. The second year's principle will be 4000 + 400, or Rs. 4400. The interest for the second year will therefore be equal to 10% of Rs. 4400, or Rs. 440. Hence, we can say that compound interest is the interest charged on interest.

Compound Interest Formula

The formula for calculating the amount received when interest is compounded annually:

Amount = Principal (1 + Rate/100)

The total compounded interest over the term is calculated as

Compound Interest = Amount - Principal

Example

1. **In how many years will an amount of Rs. 4000 will be doubled, if the interest rate is 10% per annum?**

   ***Solution**:* Let the principal amount be P, R be the rate of interest per annum, SI be simple interest, and T be the time period.

   Accordingly, SI = PRT/100

   4000 = 4000 × 10 × T (because SI = P)

   T = 40000/ 4000

   T = 10 years

Hence, the amount of INR 4000 will be doubled in 10 years.

Difference between Simple Interest and Compound Interest

Compound interest is a different kind of interest. Simple interest is based on the principal amount, but compound interest is based on the principal amount along with interest over time. This is the main distinction between simple and compound interest. To further comprehend the idea of simple interest, let's look at a straightforward example.

SAMPLE QUESTIONS ON INTEREST

Example 1: Arpit takes an Education loan of Rs 15000 from a bank for a period of 1 year. The rate of interest is 10% per annum. Find the interest and the amount he has to pay at the end of a year.

***Solution**:* Here, the loan sum = P = Rs 15000

Rate of interest per year = R = 10%

Time for which it is borrowed = T = 1 year

Thus, simple interest for a year, SI = (P × R ×T) / 100 = (15000 × 10 ×1) / 100 = Rs 1500

Amount that Ram has to pay to the bank at the end of the year = Principal + Interest = 15000 + 1500 = Rs.16,500

Example 2: Ankur borrowed Rs 100,000 for 3 years at the rate of 4.5% per annum. Find the interest accumulated at the end of 3 years.

Solution: P = Rs 100,000

R = 4.5%

T = 3 years

SI = (P × R ×T) / 100 = (100,000 × 4.5 ×3) / 100 = Rs 13,500

Example 3: Find out the difference between the compound interests on Rs. 5 Lakh for 1 years at 9% per annum compounded quarterly and half-yearly?

(A) Compounded Interest half yearly is Rs. 5460123

(B) Compounded Interest quarterly is Rs. 546541

(A) - (B) = 529

Answer is 529

PROFIT AND LOSS

Overview

Profit and loss are the one and only essential concepts that drive trade and businesses. Understanding the idea of profit and loss is crucial. Maintaining a record of one's own Income and expenses is beneficial. It is commonly believed that unless one learns what is being earned and what is being lost, understanding the concept of money can become difficult. Businessmen used profit and loss to estimate market prices for goods and understand how lucrative a business is. There is a selling price and a cost price for every goods. One can determine the profit made or loss suffered for a certain product based on the values of these prices. Parents tells their children the concept of Market Price on each item. Later, as the children grows they understand what is discount. This can be done by comparing.

So, in this section of the chapter, we'll discuss about the idea of profit and loss as well as how to calculate it.

Profit and Loss: Related Terms

Profit relates to gain; and **Loss** is the opposite of profit.

1. **Profit (P):** A product is sold at a profit if the price is higher than the cost price. For instance, if a piece of land was bought for Rs. 1,20,000 and sold for Rs. 2,20,000 four years later, there would be a profit of Rs 1 lakh.
2. **Loss (L):** When a product is sold for less than what it costs to produce, the seller suffers a loss. For instance, if a Laptop was purchased for Rs. 50,000 and sold for Rs. 35,000 a year later, the seller would have suffered a Rs 15,000 loss.
3. **Cost Price (CP):** It refers to the price at which a product is made or purchased. It can occasionally additionally cover overhead costs, transportation costs, etc. Shiva, for instance, paid Rs. 20,000 for an Air Conditioner and added Rs. 1500 for shipping and Rs. 2000 for installation. Therefore, the final cost price equals the amount of all completed expenditures, or Rs. 23,500. This cost price is divided into two more categories:
   - *Fixed Cost*: Fixed cost is constant as it does not vary with situations.
   - *Variable Cost*: It could vary depending on the situation.

4. **Selling Price (SP):** It's the price at which a product is offered for sale. It could be greater than, equal to, or lower than the item's cost price. For instance, if a store owner purchased a table for Rs. 800 and sold it for Rs 1000, the furniture's cost price is Rs 800 and its selling price is Rs 1000.

5. **Marked Price (MP):** Shop owners essentially label this to provide a discount to the customers in such a way that.,

   - Discount = Marked Price – Selling Price
   - Discount Percentage = (Discount/Marked price) x 100

6. **Profit Percent (P%):** It is the percentage of profit on the price on which the product was purchased or manufactured.

7. **Loss Percent (L%):** It is the percentage of profit on the price on which the product was purchased or manufactured..

Profit and Loss: Formulas

- The profit or gain is equal to the selling price(SP) (-) cost price(CP).

Loss is equal to the cost price (CP)(-) selling price(SP). The formula for the profit and loss percentage is:

Profit percentage (P%) = (Profit /Cost Price) x 100

Loss percentage (L%) = (Loss / Cost price) x 100

Concept of Profit and Loss: Explained with Example

- Let's use profit and loss to more easily comprehend the idea. Consider a shop owner who purchases a diary from the market for Rs. 16 and sells it at his store for Rs. 20Amount invested by the shopkeeper or Cost Price = Rs 16
- The amount received by the shopkeeper or Selling Price = Rs 20

Here,

Cost Price = Rs 16

Selling Price = Rs 20

Profit = Selling Price - Cost Price

= Rs 20 - Rs 16

= Rs 4

Therefore, the shopkeeper made a profit of Rs 4 on selling a diary.

Now, let us find what percent of profit was made by the shopkeeper.

Here,

Profit % = Profit/Cost price×100

= 4/10×100

= 20%

Thus, the shopkeeper earned a profit of 20% of the cost price.

Formulas

Profit	Loss
Cost Price (CP) < Selling Price (SP)	Cost Price (CP) > Selling Price (SP)
Profit= S.P.-C.P.	Loss= C.P.-S.P.
S.P. = C.P. + Profit	C.P. = S.P. + Loss
C.P. = S.P. - Profit	S.P. = C.P. – Loss
Profit % = $\frac{\text{Profit}}{\text{C.P.}} \times 100$	Loss % = $\frac{\text{Loss}}{\text{C.P.}} \times 100$

SAMPLE QUESTIONS ON PROFIT AND LOSS

Q. 1: Suppose a shopkeeper has bought 1 kg of Mangoes for Rs. 120. And sold it for Rs. 150 per kg. How much is the profit earned by him?

Solution:

Cost Price for Mangoes is Rs. 120

Selling Price for Mangoes is Rs. 150

Then profit gained by shopkeeper is ; P = SP – CP

P = 150 – 120 = Rs. 30/-

Q.2: Calculate the percentage of the profit gained by the shopkeeper in above situation.

Solution:

We know, Profit percentage = (Profit /Cost Price) x 100

Therefore, Profit percentage = (30/120) x 100 = 25%.

Q.3: A man buys a Cooler for Rs. 2000 and sold it at a loss of 15%. What is the selling price of the Cooler ?

Solution: Cost Price of the Cooler is Rs. 2000

Loss percentage is 15%

As we know, Loss percentage = (Loss/Cost Price) x 100

15 = (Loss/2000) x 100

Therefore, Loss = 300 Rs.

As we know, Loss = Cost Price – Selling Price

So, Selling Price = Cost Price – Loss

= 2000 – 300

Selling Price = Rs. 1700/-

Practice Questions

1. A Computer is sold at Rs. 12,050 with 15% profit. What would be the gain or loss percentage if it had been sold at Rs. 10,980?
2. Suppose the CP of 25 pencils is the same as the SP of some pencils . If the profit is 20%, then what is the number of pens sold?
3. A dealer sells goods at a 9% loss on cost price but uses 20% less weight. Compute profit or loss percentage.

List of Further Readings and References

- National Council of Educational Research and Training: https://ncert.nic.in
- National Institute of Open Schooling: https://www.nios.ac.in/

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