CHAPTER 7

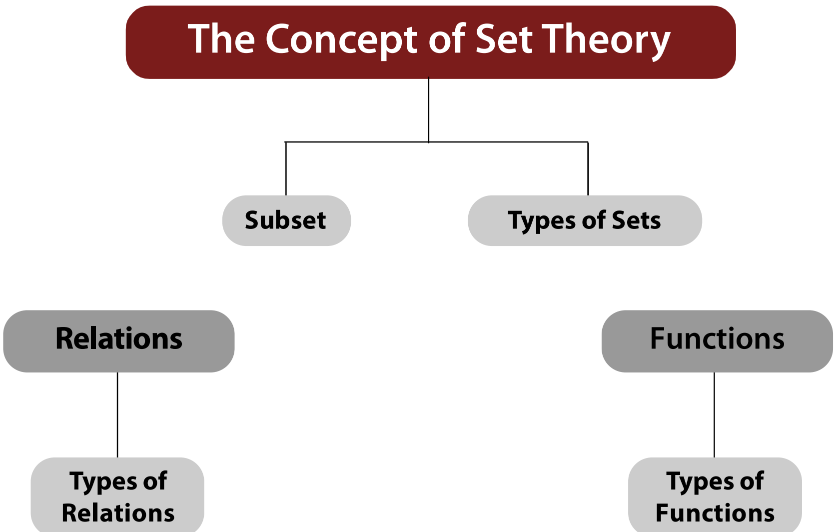
SETS, RELATIONS AND FUNCTIONS

LEARNING OBJECTIVES

After reading this chapter, students will be able to understand:

- Understand the concept of set theory.
- Appreciate the basics of functions and relations.
- Understand the types of functions and relations.
- Solve problems relating to sets, functions and relations.

CHAPTER OVERVIEW



In our mathematical language, everything in this universe, whether living or non-living, is called an object.

If we consider a collection of objects given in such a way that it is possible to tell beyond doubt whether a given object is in the collection under consideration or not, then such a collection of objects is called a *well-defined collection of objects.*

7.1 SETS

A set is defined to be a collection of well-defined distinct objects. This collection may be listed or described. Each object is called an element of the set. We usually denote sets by capital letters and their elements by small letters.

Example: A = {a, e, i, o, u}

B = {2, 4, 6, 8, 10}

C = {pqr, prq, qrp, rqp, qpr, rpq}

D = {1, 3, 5, 7, 9}

E = {1,2}

etc.

This form is called Roster or Braces form. In this form we make a list of the elements of the set and put it within braces { }.

Instead of listing we could describe them as follows :

A = the set of vowels in the alphabet

B = The set of even numbers between 2 and 10 both inclusive.

C = The set of all possible arrangements of the letters p, q and r

D = The set of odd digits between 1 and 9 both inclusive.

E = The set of roots of the equation $x^2 - 3x + 2 = 0$

Set B, D and E can also be described respectively as

B = $\{x : x = 2m$ and m being an integer lying in the interval $0 < m < 6\}$

D = $\{2x - 1 : 0 < x < 5$ and x is an integer$\}$

E = $\{x : x^2 - 3x + 2 = 0\}$

This form is called set-Builder or Algebraic form or Rule Method. This method of writing the set is called Property method. The symbol : or/reads 'such that'. In this method, we list the property or properties satisfied by the elements of the set.

We write, {x:x satisfies properties P}. This means, "the set of all those x such that x satisfies the properties P".

A set may contain either a finite or an infinite number of members or elements. When the number of members is very large or infinite it is obviously impractical or impossible to list them all. In such case.

we may write as :

N = The set of natural numbers = {1, 2, 3…..}

W = The set of whole numbers = {0, 1, 2, 3,…)

etc.

I. The members of a set are usually called elements. In A = {a, e, i, o, u}, a is an element and we write $a \in A$ i.e. a belongs to A. But 3 is not an element of B = {2, 4, 6, 8, 10} and we write $3 \notin B$. i.e. 3 does not belong to B.

II. If every element of a set P is also an element of set Q we say that P is a subset of Q. We write $P \subset Q$. Q is said to be a superset of P. For example {a, b} $\subset$ {a, b, c}, {2, 4, 6, 8, 10} $\subset$ N. If there exists even a single element in A, which is not in B then A is not a subset of B.

III. If P is a subset of Q but P is not equal to Q then P is called a proper subset of Q.

IV. Φ has no proper subset.

Illustration: {3} is a proper subset of {2, 3, 5}. But {1, 2} is not a subset of {2, 3, 5}.

Thus if P = {1, 2} and Q = {1, 2 ,3} then P is a subset of Q but P is not equal to Q. So, P is a proper subset of Q.

To give completeness to the idea of a subset, we include the set itself and the empty set. The empty set is one which contains no element. The empty set is also known as **null or void** set usually denoted by { } or Greek letter Φ , to be read as phi. For example the set of prime numbers between 32 and 36 is a null set. The subsets of {1, 2, 3} include {1, 2, 3}, {1, 2}, {1, 3}, {2, 3}, {1}, {2}, {3} and { }.

A set containing n elements has 2^n subsets. Thus a set containing 3 elements has 2^3 (=8) subsets. A set containing n elements has $2^n - 1$ proper subsets. Thus a set containing 3 elements has $2^3 - 1$ =7 subsets. The proper subsets of { 1,2,3} include {1, 2}, {1, 3}, {2, 3}, {1}, {2}, {3], { } .

Suppose we have two sets A and B. The intersection of these sets, written as $A \cap B$ contains those elements which are in A and are also in B.

For example A = {2, 3, 6, 10, 15}, B = {3, 6, 15, 18, 21, 24} and C = { 2, 5, 7}, we have $A \cap B$ = { 3, 6, 15}, $A \cap C$ = {2}, $B \cap C = \Phi$, where the intersection of B and C is empty set. So, we say B and C are disjoint sets since they have no common element. Otherwise sets are called overlapping or intersecting sets. The union of two sets, A and B, written as $A \cup B$ contain all these elements which are in either A or B or both.

So $A \cup B$ = {2, 3, 6, 10, 15, 18, 21, 24}

$A \cup C$ = {2, 3, 5, 6, 7, 10, 15}

A set which has at least one element is called non-empty set . Thus the set { 0 } is non-empty set. It has one element say 0.

Singleton Set: A set containing one element is called Singleton Set.

For example {1} is a singleton set, whose only element is 1.

Equal Set: Two sets A & B are said to be equal, written as A = B if every element of A is in B and every element of B is in A.

Illustration: If A = {2, 4, 6} and B = {2, 4, 6} then A = B.

Remarks : (I) The elements of the two sets may be listed in any order.

Thus, {1, 2, 3} = {2, 1, 3} = {3, 2, 1} etc.

(II) The repetition of elements in a set is meaningless.

Example: {x : x is a letter in the word "follow"} = {f, o, l, w}

Example: Show that Φ, {0} and 0 are all different.

Solution: Φ is a set containing no element at all; {0} is a set containing one element, namely 0. And 0 is a number, not a set.

Hence Φ ,{0} and 0 are all different.

The set which contains all the elements under consideration in a particular problem is called *the universal set* denoted by S. Suppose that P is a subset of S. Then the complement of P, written as P^c (or P') contains all the elements in S but not in P. This can also be written as S – P or S ~ P. S – P = $\{x : x \in S, x \notin P\}$.

For example let S = {0, 1, 2, 3, 4, 5, 6, 7, 8, 9}

P = {0, 2, 4, 6, 8}

Q = {1, 2, 3, 4, 5)

Then P' = {1, 3 ,5 ,7, 9} and Q'= {0 , 6 , 7, 8, 9}

Also $P \cup Q$ = {0, 1, 2, 3, 4, 5, 6, 8}, $(P \cup Q)'$ = {7, 9}

$P \cap Q$ = {2, 4}

$P \cup Q'$ = {0, 2, 4, 6, 7, 8, 9}, $(P \cap Q)'$ = {0, 1, 3, 5, 6, 7, 8, 9}

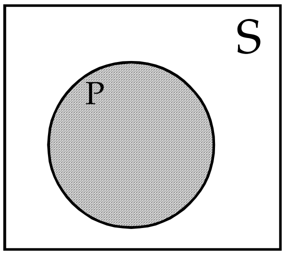
$P' \cup Q'$ = {0, 1, 3, 5, 6, 7, 8, 9}

$P' \cap Q'$ = {7, 9}

So it can be noted that $(P \cup Q)' = P' \cap Q'$ and $(P \cap Q)' = P' \cup Q'$. These are known as De Morgan's laws.

7.2 VENN DIAGRAMS

We may represent the above operations on sets by means of Euler - Venn diagrams.



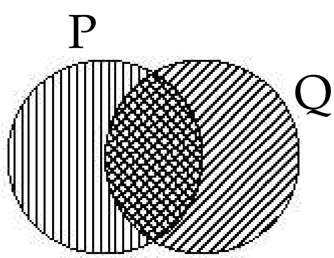


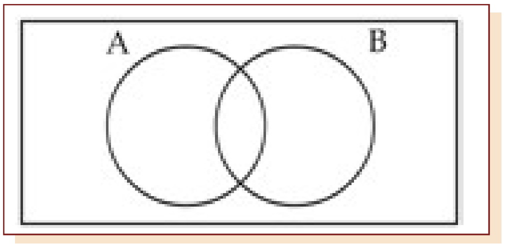
Fig. 1

Thus Fig. 1(a) shows a universal set S represented by a rectangular region and one of its subsets P represented by a circular shaded region.

The un-shaded region inside the rectangle represents P'.

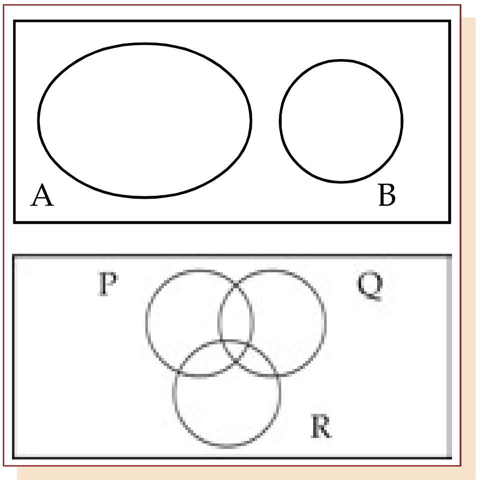
Figure 1(b) shows two sets P and Q represented by two intersecting circular regions. The total shaded area represents PUQ, the cross-hatched "intersection" represents $P \cap Q$.

The number of distinct elements contained in a finite set A is called its **cardinal number**. It is denoted by n(A). For example, the number of elements in the set R = {2, 3, 5, 7} is denoted by n(R). This number is called the cardinal number of the set R.



Thus $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

If A and B are disjoint sets, then $n(A \cup B) = n(A) + n(B)$ as $n(A \cap B) = 0$



For three sets P, Q and R

$n(P \cup Q \cup R) = n(P) + n(Q) + n(R) - n(P \cap Q) - n(Q \cap R) - n(P \cap R) + n(P \cap Q \cap R)$

When P, Q and R are disjoint sets

$= n(P) + n(Q) + n(R)$

Illustration: If A = {2, 3, 5, 7} , then n(A) = 4

Equivalent Set: Two finite sets A & B are said to be equivalent if n (A) = n(B).

Clearly, **equal sets are equivalent but equivalent sets need not be equal.**

Illustration: The sets A = {1, 3, 5} and B = {2, 4, 6} are equivalent but not equal.

Here n (A) = 3 = n (B) so they are equivalent sets. But the elements of A are not in B. Hence they are not equal though they are equivalent.

Power Set : The collection of all possible subsets of a given set A is called the power set of A, to be denoted by P(A).

Illustration: (i) If A = {1, 2, 3} then

P(A) = { {1, 2, 3}, {1, 2}, {1, 3}, {2, 3}, {1}, {2}, {3}, Φ }

(ii) If A = {1, {2}}, we may write A = {1, B} when B = {2} then

P(A) = {Φ , {1}, {B }, {1, B}} = {Φ , {1}, {{2}}, {1,{2}}}

EXERCISE 7 (A)

Choose the most appropriate option or options (a) (b) (c) or (d).

1. The number of subsets of the set {2, 3, 5} is

   (a) 3 (b) 8 (c) 6 (d) none of these

2. The number of subsets of a set containing n elements is

   (a) 2^n (b) 2^{-n} (c) n (d) none of these

3. The null set is represented by

   (a){Φ } (b) { 0 } (c) Φ (d) none of these

4. A = {2, 3, 5, 7} , B = { 4, 6, 8, 10} then $A \cap B$ can be written as

   (a) { } (b) {Φ} (c) (AUB)' (d) None of these

5. The set {x|0<x<5} represents the set when x may take integral values only

   (a) {0, 1, 2, 3, 4, 5} (b) {1, 2, 3, 4} (c) {1, 2, 3, 4, 5 } (d) none of these

6. The set {0, 2, 4, 6, 8, 10} can be written as

   (a) {2x | 0<x <5} (b) {x : 0<x<5} (c) {2x : $0 \le x \le 5$} (d) none of these

From Q.7 to Q.10 The data to be used If P = {1, 2, 3, 5, 7}, Q = {1, 3, 6, 10, 15},
Universal Set S = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15}

7. The cardinal number of $P \cap Q$ is

   (a) 3 (b) 2 (c) 0 (d) none of these

8. The cardinal number of $P \cup Q$ is

   (a) 10 (b) 9 (c) 8 (d) none of these

9. n (P') is

   (a) 10 (b) 5 (c) 6 (d) none of these

10. n(Q') is

    (a) 4 (b) 10 (c) 4 (d) none of these

11. The set of cubes of the natural number is

    (a) a finite set (b) an infinite set (c) a null set (d) none of these

12. The set $\{2^x \mid x$ is any positive rational number$\}$ is
(a) an infinite set (b) a null set (c) a finite set (d) none of these

13. $\{1-(-1)^x\}$ for all integer x is the set
(a) $\{0\}$ (b) $\{2\}$ (c) $\{0, 2\}$ (d) none of these

14. E is a set of positive even numbers and O is a set of positive odd numbers, then $E \cup O$ is a
(a) set of whole numbers (b) N (c) a set of rational number (d) none of these

15. If R is the set of positive rational numbers and E is the set of real numbers then
(a) $R \subseteq E$ (b) $R \subset E$ (c) $E \subset R$ (d) none of these

16. If N is the set of natural numbers and I is the set of positive integers, then
(a) N = I (b) $N \subset I$ (c) $N \subseteq I$ (d) none of these

17. If I is the set of isosceles triangles and E is the set of equilateral triangles, then
(a) $I \subset E$ (b) $E \subset I$ (c) E = I (d) none of these

18. If R is the set of isosceles right angled triangles and I is set of isosceles triangles, then
(a) R = I (b) $R \supset I$ (c) $R \subset I$ (d) none of these

19. $\{n(n+1)/2 : n$ is a positive integer$\}$ is
(a) a finite set (b) an infinite set (c) is an empty set (d) none of these

20. If A = {1, 2, 3, 5, 7}, and $B = \{x^2 : x \in A\}$
(a) n(b) = n(A) (b) n(B) > n(A) (c) n(A)= n(B) (d) n(A)<n(B)

21. $A \cup A$ is equal to
(a) A (b) E (c) ϕ (d) none of these

22. $A \cap A$ is equal to
(a) ϕ (b) A (c) E (d) none of these

23. $(A \cup B)'$ is equal to
(a) $(A \cap B)'$ (b) $A \cup B'$ (c) $A' \cap B'$ (d) none of these

24. $(A \cap B)'$ is equal to
(a) $(A' \cup B)'$ (b) $A' \cup B'$ (c) $A' \cap B'$ (d) none of these

25. $A \cup E$ is equal to (E is a superset of A)
(a) A (b) E (c) ϕ (d) none of these

26. $A \cap E$ is equal to (E is a superset of A)
(a) A (b) E (c) ϕ (d) none of these

27. $E \cup E$ is equal to E is a superset of A

(a) E (b) ϕ (c) 2E (d) none of these

28. $A \cap E'$ is equal to E is a superset of A

(a) E (b) ϕ (c) A (d) none of these

29. $A \cap \phi$ is equal to E is a superset of A

(a) A (b) E (c) ϕ (d) none of these

30. AUA' is equal to E is a superset of A

(a) E (b) ϕ (c) A (d) none of these

31. If E = {1, 2, 3, 4, 5, 6, 7, 8, 9}, the subset of E satisfying 5 + x > 10 is

(a) {5, 6, 7, 8, 9} (b) {6, 7, 8, 9} (c) {7, 8, 9} (d) none of these

32. If $A\Delta B = (A-B) \cup (B-A)$ and A = {1, 2, 3, 4}, B = {3,5,7} than $A\Delta B$ is

(a) {1, 2, 4, 5, 7} (b) {3} (c) {1, 2, 3, 4, 5, 7} (d) none of these

[Hint : If A and B are any two sets, then

$A - B = \{ x : x \in A, x \notin B\}$.

[i.e. A - B Contains all elements of A but not in B] .

7.3 PRODUCT OF SETS

Ordered Pair : Two elements a and b, listed in a specific order, form an ordered pair, denoted by (a, b).

Cartesian Product of sets : If A and B are two non-empty sets, then the set of all ordered pairs (a, b) such that a belongs to A and b belongs to B , is called the Cartesian product of A and B, to be denoted by A × B.

Thus, $A \times B = \{(a, b) : a \in A \text{ and } b \in B\}$

If $A = \Phi$ or $B = \Phi$, we define $A \times B = \Phi$

Illustration: Let A = {1, 2, 3}, B = {4, 5}

Then A × B = { (1, 4), (1, 5), (2, 4) (2, 5), (3, 4), (3, 5) }

Example: If A × B = { (3, 2) , (3, 4) , (5, 2), (5, 4) } , find A and B.

Solution: Clearly A is the set of all first co-ordinates of A × B, while B is the set of all second co-ordinates of elements of A × B.

Therefore A = {3, 5} and B = {2 , 4}

Example: Let P = {1, 3, 6} and Q {3, 5}

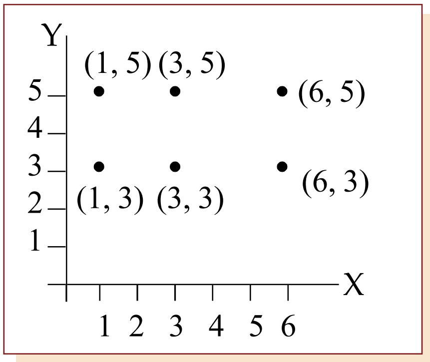
The product set P × Q = {(1, 3), (1, 5), (3, 3), (3, 5), (6, 3), (6, 5)} .

Notice that n(P×Q) = n(P) × n(Q) and ordered pairs (3, 5) and (5, 3) are not equal. and Q × P = {(3, 1), (3, 3), (3, 6), (5, 1), (5, 3), (5, 6)}

So P × Q ≠ Q × P; but n (P × Q) = n (Q × P).

Illustration: Here n(P) = 3 and n(Q) = 2, n (P × Q) = 6. Hence n(P×Q) = n(P) × n(Q). and n(P × Q) = n(Q × P) = 6.

We can represent the product set of ordered pairs by plotting points in the XY plane.



If X=Y= the set of all natural numbers, the product set XY represents an infinite equal lattice of points in the first quadrant of the XY plane.

7.4 RELATIONS AND FUNCTIONS

Any subset of the product set X.Y is said to define a **relation** from X to Y and any relation from X to Y in which no two different ordered pairs have the same first element is called a **function**.

Let A and B be two non-empty sets. Then, a rule or a correspondence f which associates to each element x of A, a unique element, denoted by f(x) of B , is called a function or **mapping** from A to B and we write f : A→B

The element f(x) of B is called the image of x, while x is called the pre-image of f (x).

7.5 DOMAIN & RANGE OF A FUNCTION

Let f : A→B, then A is called the domain of f, while B is called the co-domain of f.

The set f(A) = {f (x) : x ∈ A} is called the range of f.

Illustration: Let A = {1, 2, 3, 4} and B ={1, 4, 9, 16, 25}

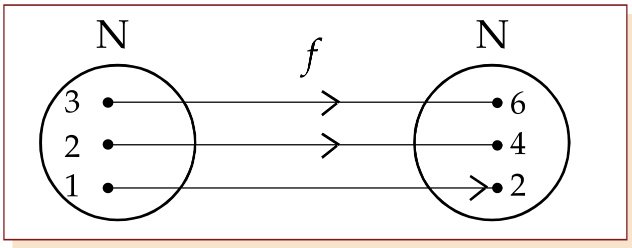
We consider the rule $f(x) = x^2$. Then f(1) = 1 ; f(2) =4 ; f (3) = 9 & f (4) = 16.

Then clearly each element in A has a unique image in B.

So, f : A → B : $f(x) = x^2$ is a function from A to B.

Here domain of f = {1, 2, 3, 4} and range of f = {1, 4, 9, 16}

Example: Let N be the set of all natural numbers. Then, the rule



$f : N \rightarrow N : f(x) = 2x$, for all $x \in N$

is a function from N to N, since twice a natural number is unique.

Now, f (1) = 2; f (2) = 4; f(3) = 6 and so on.

Here domain of f = N = {1, 2 ,3, 4 ,………..} ; range of f = { 2, 4, 6,………..}

This may be represented by the mapping diagram or arrow graph.

7.6 VARIOUS TYPES OF FUNCTIONS

One-One Function : Let $f : A \rightarrow B$. If different elements in A have different images in B, then f is said to be a one-one or an injective function or mapping.

Illustration: (i) Let A = {1, 2, 3} and B = {2, 4, 6}

Let us consider $f : A \rightarrow B : f(x) = 2x$.

Then f(1) = 2; f(2) =4 ; f (3) = 6

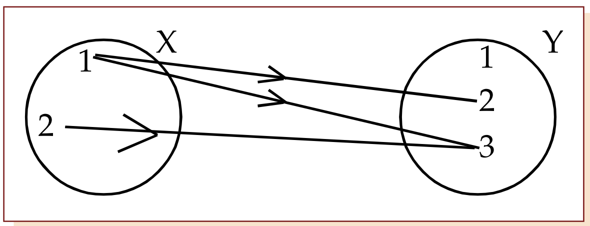
Clearly, f is a function from A to B such that different elements in A have different images in B. Hence f is one-one.

Remark : Let $f : A \rightarrow B$ and let $x_1 , x_2 \in A$.

Then $x_1 = x_2$ implies $f(x_1) = f(x_2)$ is always true.

But **$f(x_1) = f(x_2)$ implies $x_1 = x_2$ is true only when f is one-one.**

(ii) let x={1, 2, 3, 4} and y={1, 2, 3}, then the subset {(1, 2), (1, 3), (2, 3)} defines a relation on X.Y.



Notice that this particular subset contains all the ordered pair in X.Y for which the X element (x) is less than the Y element (y). So in this subset we have X<Y and the relation between the set, is "less than". This relation is not a function as it includes two different ordered pairs (1, 2), (1, 3) have same first element.

X.Y={(1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (2, 3)

(3, 1), (3, 2), (3, 3),(4, 1), (4, 2), (4, 3)}

The subset {(1, 1), (2, 2), (3, 3)} defines the function y= x as different ordered pairs of this subset have different first element.

Onto or Surjective Functions : Let f : A→B. If every element in B has at least one pre-image in A, then f is said to be an onto function.

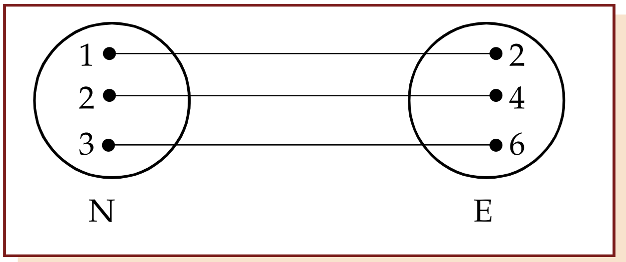
If f is onto, then corresponding to each y $\in$ B, we must be able to find at least one element x $\in$ A such that y = f (x)

Clearly, f is onto if and only if range of f = B

Illustration: Let N be the set of all natural numbers and E be the set of all even natural numbers. Then, the function

f : N → E : f (x) = 2x , for all x $\in$ N

is onto, since each element of E is of the form 2x, where x $\in$ N and the same is the f-image of x $\in$ N.



Represented on a mapping diagram it is a on to mapping of X onto Y.

Bijection Function : A one-one and onto function is said to be bijective.
A bijective function is also known as a one-to-one correspondence.

Identity Function : Let A be a non-empty set. Then, the function I defined by

I : A → A : I (x) = x for all x $\in$ A is called an identity function on A.

It is a one-to-one and onto function with domain A and range A.

Into Functions: Let f : A → B. There exists even a single element in B having no pre-image in A, then f is said to be an into function.

Illustration: Let A = {2, 3, 5, 7} and B = {0, 1, 3, 5, 7} . Let us consider f : A → B;

f (x) = x – 2. Then f(2) = 0; f(3) = 1 ; f (5) =3 & f(7) = 5.

It is clear that f is a function from A to B .

Here there exists an element 7 in B, having no pre-mage in A.

So, f is an into function.

Constant Function: Let f : A → B, defined in such a way that all the elements in A have the same image in B, then f is said to be a constant function.

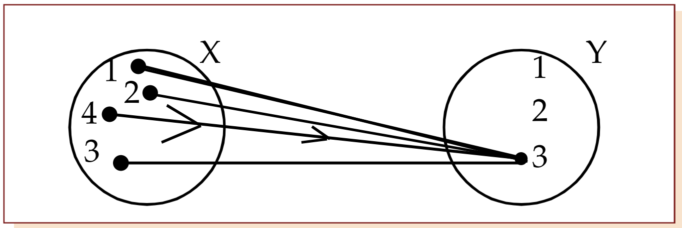
Illustration: Let A = {1, 2, 3} and B = {5, 5, 5} or B={5}. Let f : A → B : f (x) = 5 for all x ∈ A.

Then, all the elements in A have the same image namely 5 in B.

So, f is a constant function.

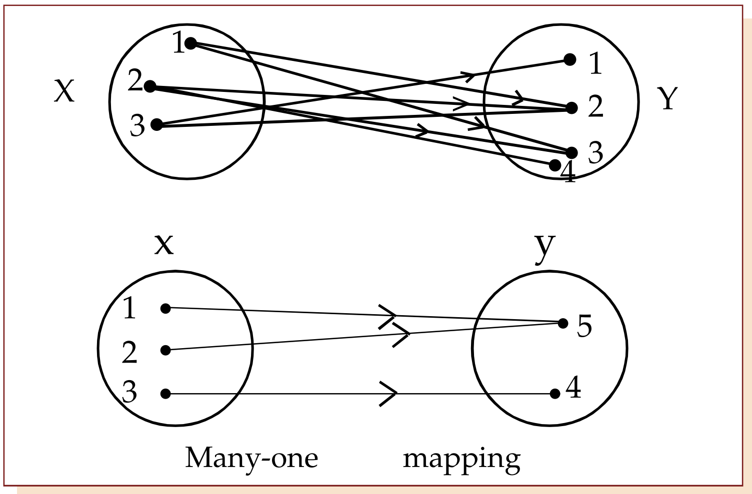
Remark: The range set of a constant function is a singleton set.

Example: Another subset of X.Y is {(1, 3), (2, 3), (3, 3), (4, 3)}



This relation is a function (a constant function). It is represented on a mapping diagram and is a many-one mapping of X into Y.

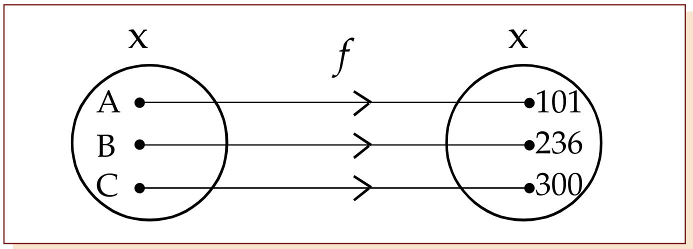
Let us take another subset {(4, 1), (4, 2), (4, 3)} of X.Y. This is a relation but not a function. Here different ordered pairs have same first element so it is not a function.



There are two example of many-one mapping.

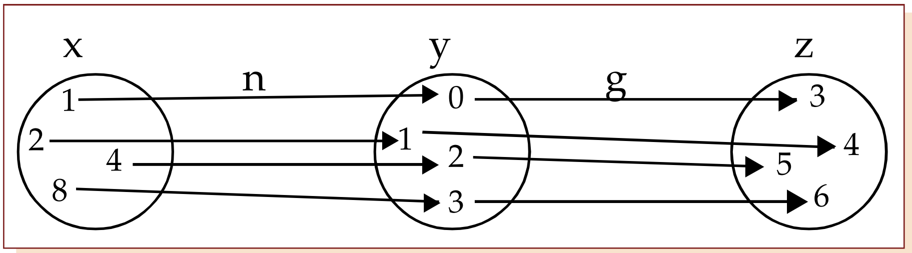
Equal Functions: Two functions f and g are said to be equal, written as f = g if they have the same domain and they satisfy the condition f(x) = g(x), for all x.

A function may simply pair people and the corresponding seat numbers in a theatre. It may simply associate weights of parcels with portal delivery charge or it may be the operation of squaring, adding to doubling, taking the log of etc.



The function f here assigning a locker number to each of the persons A, B and C. Names are associated with or mapped onto, locker numbers under the function f.

We can write $f : X \rightarrow Y$ or $f(x) = y$ or $f(B) = 236$



This diagram shows the effect of two functions n and g on the sets X, Y and Z

$n : X \rightarrow Y$ and $g : Y \rightarrow Z$

If x, y, z are corresponding elements of X, Y and Z, we write n(x) = y and g(y) = z

Thus g n(1) = 0 and g (0) = 3, so that g (n(1)) = g(0) = 3 we can write it as

g (n(1)) or g o n (1) = 3 But g(1) = 4 and n(g(1))= n(4) = 2

So $g(n) \neq n(g)$ [(or, $g (o\ n) \neq n (o\ g)$)]

The function g(n) or n(g) is called a composite function. As n(8) = 3, we say that 3 is the image of 8 under the mapping (or function) n.

Inverse Function: Let f be a one-one onto function from A to B. Let y be an arbitrary element of B. Then f being onto, there exists an element x in A such that f (x) = y.

As f is one-one this x is unique.

Thus for each $y \in B$, there exists a unique element $x \in A$ such that f (x) = y.

So, we may define a function, denoted by f^{-1} as:

$f^{-1} : B \rightarrow A : f^{-1} (y) = x$ if and only if f (x) = y.

The above function f^{-1} is called the inverse of f.

A function is invertible if and only if f is one-one onto.

Remarks: If f is one-one onto then f^{-1} is also one-one onto.

Example: net A = {1, 3, 5, 7} and B = {3, 4, 9} So, f; A→B given by {(1, 2), (3, 3), (5, 4) (7, 9)} and it is one-one on to mapping. Now if then is there of f.

If $f : A \rightarrow B$ then $f^{-1} : B \rightarrow A$.

EXERCISE 7 (B)

Choose the most appropriate option/options (a) (b) (c) or (d).

1. If A = {x, y, z}, B = {p, q, r, s} which of the relations on A to B are functions.
   (a) {(x, p), (x, q), (y, r), (z, s)}, (b) {(x, s), (y, s), (z, s)}
   (c) {(y, p), (y, q), (y, r),(z, s), (d) {(x, p), (y, r), (z, s)}
2. {(x, y) | x+y = 5} where x, y $\in$ R is a
   (a) not a function (b) a composite function (c) one-one mapping (d) none of these
3. {(x, y) | x = 4} where x, y $\in$ R is a
   (a) not a function (b) function (c) one-one mapping (d) none of these
4. {(x, y), y=x^2} where x, y $\in$ R is
   (a) not a function (b) a function (c) inverse mapping (d) none of these
5. {(x, y) | x<y} where x, y $\in$ R is
   (a) not a function (b) a function (c) one-one mapping (d) none of these
6. The domain of {(1, 7), (2, 6)} is
   (a) {1, 6} (b) {7, 6} (c) {1, 2} (d) {6, 7}
7. The range of {(3, 0), (2, 0), (1, 0), (0, 0)} is
   (a) {0, 0} (b) {0} (c) {0, 0, 0, 0} (d) none of these
8. The domain and range of {(x,y) : y = x^2} where x, y $\in$ R is
   (a) (reals, natural numbers) (b) (reals, non-negative reals)
   (c) (reals, reals) (d) none of these
9. Let the domain of x be the set {1}. Which of the following functions gives values equal to 1
   (a) f(x) = x^2, g(x) = x (b) f(x) = x, g(x) = 1–x
   (c) f(x) = x^2 + x + 2, g(x) = $(x+ 1)^2$ (d) none of these
10. If f(x) = 1/1–x, f(–1) is
   (a) 0 (b) ½ (c) 0 (d) none of these
11. If g(x) = (x–1)/x, g(–½) is
   (a) 1 (b) 2 (c) 3/2 (d) 3
12. If f(x) = 1/1–x and g(x) = (x–1)/x, then fog(x) is
   (a) x (b) 1/x (c) –x (d) none of these
13. If f(x) = 1/1–x and g(x) = (x–1)/x, then gof(x) is
   (a) x–1 (b) x (c) 1/x (d) none of these

14. The function $f(x) = 2^x$ is

(a) one-one mapping (b) one-many

(c) many-one (d) none of these

15. The range of the function $f(x) = \log_{10}(1 + x)$ for the domain of real values of x when $0 \leq x \leq 9$ is

(a) [0, 1] (b) [0, 1, 2] (c) {0, 1} (d) none of these

16. The Inverse function f^{-1} of $f(x) = 2x$ is

(a) 1/2x (b) $\frac{x}{2}$ (c) 1/x (d) none of these

17. If $f(x) = x+3$, $g(x) = x^2$, then fog(x) is

(a) $x^2 + 3$ (b) $x^2 + x + 3$ (c) $(x + 3)^2$ (d) none of these

18. If $f(x) = x+3$, $g(x) = x^2$, then f(x).g(x) is

(a) $(x + 3)^2$ (b) $x^2 + 3$ (c) $x^3 + 3x^2$ (d) none of these

19. The Inverse h^{-1} when $h(x) = \log_{10}x$ is

(a) $\log_{10}x$ (b) 10^x (c) $\log_{10}(1/x)$ (d) none of these

20. For the function $h(x) = 10^{1+x}$ the domain of real values of x where $0 \leq x \leq 9$, the range is

(a) $10 \leq h(x) \leq 10^{10}$ (b) $0 \leq h(x) \leq 10^{10}$ (c) $0 < h(x) < 10$

(d) none of these

Different types of relations

Let S = {a, b, c, ….} be any set then the relation R is a subset of the product set S×S

i) If R contains all ordered pairs of the form (a, a) in S×S, then R is called reflexive. In a *reflexive* relation 'a' is related to itself .

For example, 'Is equal to' is a reflexive relation for a = a is true.

ii) If $(a, b) \in R \Rightarrow (b, a) \in R$ for every $a, b \in S$ then R is called symmetric

For example $a=b \Rightarrow b = a$. Hence the relation 'is equal to' is a symmetric relation.

iii) If $(a, b) \in R$ and $(b, c) \in R \Rightarrow (a, c) \Rightarrow R$ for every a, b, c, $\in$ S then R is called *transistive.*

For example a =b, $b=c \Rightarrow a=c$. Hence the relation 'is equal to' is a transitive relation.

A relation which is reflexive, symmetric and transitive is called an *equivalence relation* or simply an *equivalence*. 'is equal to' is an equivalence relation.

Similarly, the relation " is parallel to " on the set S of all straight lines in a plane is an equivalence relation.

Illustration: The relation "is parallel to" on the set S is

(1) reflexive, since $a \parallel a$ for $a \in S$

(2) symmetric, since a || b $\Rightarrow$ b || a

(3) transitive, since a || b , b || c $\Rightarrow$ a || c

Hence it is an equivalence relation.

Domain & Range of a relation : If R is a relation from A to B, then the set of all first co-ordinates of elements of R is called the domain of R, while the set of all second co-ordinates of elements of R is called the range of R.

So, Dom (R) = { a : (a, b) $\in$ R } & Range (R) = { b : (a, b) $\in$ R}

Illustration: Let A = {1, 2, 3} and B = {2, 4, 6}

Then A × B = {(1, 2), (1, 4), (1, 6), (2, 2), (2, 4), (2, 6), (3, 2), (3, 4), (3, 6)}

By definition every subset of A × B is a relation from A to B.

Thus, if we consider the relation

R = { (1, 2), (1, 4), (3, 2), (3, 4)} then Dom (R) = {1, 3} and Range (R)= {2, 4}

From the product set X. Y = {(1, 3), (2, 3), (3, 3), (4, 3), (2, 2), (3, 2), (4, 2), (1, 1), (2, 1), (3, 1), (4, 1)}, the subset {(1, 1), (2, 2), (3, 3)} defines the relation 'Is equal to' , the subset {(1, 3), (2, 3), (1, 2)} defines 'Is less than', the subset {(4, 3), (3, 2), (4, 2), (2, 1), (3, 1), (4, 1)} defines 'Is greater than' and the subset {(4, 3), (3, 2), (4, 2), (2, 1), (3, 1), (4, 1), (1, 1), (2, 2) (3, 3)} defines to greater 'In greater than or equal'.

Illustration: Let A = {1, 2, 3} and b = {2, 4, 6}

Then A × B = {(1, 2), (1, 4), (1, 6), (2, 2), (2, 4), (2, 6), (3, 2), (3, 4), (3, 6)}

If we consider the relation = {(1, 2), (1, 4), (3, 4)} then Dom (R) = {1, 3} and Range = {2, 4}. Here the relation "Is less than".

Identity Relation: The relation I = {(a, a) : a $\in$A} is called the identity relation on A.

Illustration: Let A = {1, 2, 3} then I = {(1, 1), (2, 2), (3, 3)}

Inverse Relation: If R be a relation on A, then the relation R^{-1} on A, defined by

R^{-1} = {(b, a) : (a, b)$\in$ R} is called an inverse relation on A.

Clearly , Dom (R^{-1}) = Range (R) & Range (R^{-1}) = Dom (R).

Illustration: Let A = {1, 2, 3} and R = {(1, 2), (2, 2), (3, 1), (3, 2)}

Then R being a subset of a × a, it is a relation on A. Dom (R) = {1, 2, 3} and Range (R) = {2, 1}

Now, R^{-1} = {(2, 1), (2, 2), (1, 3), (2, 3)}. Here, Dom (R^{-1}) = {2, 1} = Range (R) and

Range (R^{-1}) = {1, 2, 3} = Dom (R).

Illustration: Let A = {1, 2, 3}, then

(i) R1 = {(1, 1), (2, 2), (3, 3), (1, 2)}

Is reflexive and transitive but not symmetric, since (1, 2)$\in$ R_1 but (2, 1) does not belongs to R_1.

(ii) R2 = {(1, 1), (2, 2), (1, 2), (2, 1)}

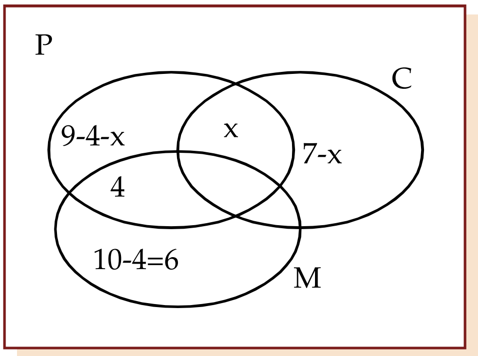
is symmetric and transitive but not reflexive, since (3, 3) does not belong to R_2.

(iii) R3 = {(1, 1), (2, 2), (3, 3), (1, 2), (2, 1), (2, 3), (3, 2)}

is reflexive and symmetric but not transitive , since (1, 2) $\in$ R3 & (2, 3) $\in$ R3 but (1, 3) does not belong to R3.

Problems and solution using Venn Diagram

1. Out of a group of 20 teachers in a school, 10 teach Mathematics, 9 teach Physics and 7 teach Chemistry. 4 teach Mathematics and Physics but none teach both Mathematics and Chemistry. How many teach Chemistry and Physics? How many teach only Physics ?



Let x be the no. of teachers who teach both Physics & Chemistry.

9 – 4 – x + 6 + 7 – x + 4 + x = 20

or 22 – x = 20

or x = 2

Hence, 2 teachers teach both Physics and Chemistry and 9 – 4 – 2 = 3 teachers teach only Physics.

2. A survey shows that 74% of the Indians like grapes, whereas 68% like bananas.

What percentage of the Indians like both grapes and bananas?

Solution: Let P & Q denote the sets of Indians who like grapes and bananas respectively. Then n(P) = 74, n(Q) = 68 and n(P $\cup$ Q) = 100.

We know that n(P $\cap$ Q) = n(P) + n(Q) – n(P $\cup$ Q) = 74 + 68 – 100 = 42.

Hence, 42% of the Indians like both grapes and bananas.

3. In a class of 60 students, 40 students like Maths, 36 like Science, and 24 like both the subjects. Find the number of students who like

(i) Maths only (ii) Science only (iii) either Maths or Science

(iv) neither Maths nor Science

Solution: Let M = students who like Maths and S = students who like Science

Then n(M) = 40, n(S) = 36 and n $(M \cap S) = 24$

Hence,

(i) $n(M) - n(M \cap S) = 40 - 24 = 16$ = number of students like Maths only.

(ii) $n(S) - n(M \cap S) = 36 - 24 = 12$ = number of students like Science only.

(iii) $n(M \cup S) = n(M) + n(S) - n(M \cap S) = 40 + 36 - 24 = 52$ = number of students who like either Maths or Science.

(iv) $n(M \cup S)^c = 60 - n(M \cup S) = 60 - 52 = 8$ = number of students who like neither Maths nor Science.

SUMMARY

- A set is defined to be a collection of well-defined distinct objects. This collection may be listed or described. Each object is called an element of the set. We usually denote sets by capital letters and their elements by small letters.
- **Singleton Set:** A set containing one element is called Singleton.
- **Equal Set:** Two sets A & B are said to be equal, written as A = B if every element of A is in B and every element of B is in A.
- **Universal Set:** The set which contains all the elements under consideration in a particular problem is called *the universal set* denoted by S. Suppose that P is a subset of S. Then the complement of P, written as P^c (or P') contains all the elements in S but not in P. This can also be written as S – P or S ~ P. $S - P = \{x : x \in S, x \notin P\}$.
- If A and B are two sets then

  $n(AUB) = n(A) + n(B) - n(A \cap B)$
- If A and B are disjoint sets, then $n(AUB) = n(A) + n(B)$ as $n(A \cap B) = 0$
- For three sets P, Q and R

  $n(PUQUR) = n(P) + n(Q) + n(R) - n(P \cap Q) - n(Q \cap R) - n(P \cap R) + n(P \cap Q \cap R)$

  When P, Q and R are disjoint sets

  $= n(P) + n(Q) + n(R)$
- **Equivalent Set:** Two finite sets A & B are said to be equivalent if n (A) = n(B).
- **Power Set:** The collection of all possible subsets of a given set A is called the power set of A, to be denoted by P(A).

  1. A set containing n elements has 2^n subsets.
  2. A set containing n elements has 2^n-1 proper subsets

- **Ordered Pair:** Two elements a and b, listed in a specific order, form an ordered pair, denoted by (a, b).
- **Cartesian Product of sets:** If A and B are two non-empty sets, then the set of all ordered pairs (a, b) such that a belongs to A and b belongs to B , is called the Cartesian product of A and B, to be denoted by A × B.

  Thus, $A \times B = \{(a, b) : a \in A \text{ and } b \in B\}$

  If $A = \Phi$ or $B = \Phi$, we define $A \times B = \Phi$
- **Relation and Function:** Any subset of the product set X.Y is said to define a **relation** from X to Y and any relation from X to Y in which no two different ordered pairs have the same first element is called a **function**.

  Let A and B be two non-empty sets. Then, a rule or a correspondence f which associates to each element x of A, a unique element, denoted by f(x) of B , is called a function or **mapping** from A to B and we write $f : A \rightarrow B$

  The element f(x) of B is called the image of x, while x is called the pre-image of f (x).

  Let $f : A \rightarrow B$, then A is called the domain of f, while B is called the co-domain of f.

  The set $f(A) = \{ f(x) : x \in A \}$ is called the range of f.
- **One-one Function:** Let $f : A \rightarrow B$. If different elements in A have different images in B, then f is said to be a one-one or an injective function or mapping.
- **Onto or Surjective Functions:** Let $f : A \rightarrow B$. If every element in B has at least one pre-image in A, then f is said to be an onto function.

  If f is onto, then corresponding to each $y \in B$, we must be able to find at least one element $x \in A$ such that $y = f(x)$

  Clearly, f is onto if and only if range of f = B
- **Bijection Function:** A one-one and onto function is said to be bijective.

  A bijective function is also known as a one-to-one correspondence.
- **Identity Function:** Let A be a non-empty set . Then, the function I defined by

  $I : A \rightarrow A : I(x) = x$ for all $x \in A$ is called an identity function on A.
- It is a one-to-one onto function with domain A and range A.

  Into Functions: Let $f : A \rightarrow B$. There exists even a single element in B having no pre-image in A, then f is said to be an into function.
- **Constant Function:** Let $f : A \rightarrow B$, defined in such a way that all the elements in A have the same image in B, then f is said to be a constant function.
- **Equal Functions:** Two functions f and g are said to be equal, written as f = g if they have the same domain and they satisfy the condition f(x) = g(x), for all x.
- **Inverse Function:** Let f be a one-one onto function from A to B. Let y be an arbitrary element of B. Then f being onto, there exists an element x in A such that f (x) = y.

A function is invertible if and only if f is one-one onto.

- **Different types of relations:**

Let S = {a, b, c, ...} be any set then the relation R is a subset of the product set S×S

i) If R contains all ordered pairs of the form (a, a) in S×S, then R is called reflexive. In a *reflexive* relation 'a' is related to itself.

For example, 'Is equal to' is a reflexive relation for a = a is true.

ii) If (a, b) $\in$ R $\Rightarrow$ (b, a) $\in$ R for every a, b $\in$ S then R is called symmetric

For Example a = b $\Rightarrow$ b = a. Hence the relation 'is equal to' is a symmetric relation.

iii) If (a, b) $\in$ R and (b, c) $\in$ R $\Rightarrow$ (a, c) $\Rightarrow$ R for every a, b, c, $\in$ S then R is called *transistive.*

For Example a = b, b = c $\Rightarrow$ a = c. Hence the relation 'is equal to' is a transitive relation.

A relation which is reflexive, symmetric and transitive is called an *equivalence relation* or simply an *equivalence*. 'is equal to' is an equivalence relation.

Similarly, the relation "is parallel to" on the set S of all straight lines in a plane is an equivalence relation.

- **Domain & Range of a relation:** If R is a relation from A to B, then the set of all first co-ordinates of elements of R is called the domain of R, while the set of all second co-ordinates of elements of R is called the range of R.

So, Dom (R) = {a : (a, b) $\in$ R} & Range (R) = {b : (a, b) $\in$ R}

EXERCISE 7 (C)

Choose the most appropriate option/options (a) (b) (c) or (d).

1. "Is smaller than" over the set of eggs in a box is

   (a) Transitive (T) (b) Symmetric (S) (c) Reflexive (R) (d) Equivalence (E)

2. "Is equal to" over the set of all rational numbers is

   (a) (T) (b) (S) (c) (R) (d) E

   [By using using R = Reflexive; T = Transitive, S = Symmetric and E = Equivalence from Q.No. 2 to 8]

3. "has the same father as" ...... over the set of children

   (a) R (b) S (c) T (d) E

4. "is perpendicular to" over the set of straight lines in a given plane is

   (a) R (b) S (c) T (d) E

5. "is the reciprocal of" .... over the set of non-zero real numbers is

   (a) S (b) R (c) T (d) none of these

6. $\{(x, y)/x \in x, y \in y, y = x\}$ is

(a) R (b) S (c) T (d) E

7. $\{(x,y) / x + y = 2x$ where x and y are positive integers$\}$, is

(a) R (b) S (c) T (d) E

8. "Is the square of" over n set of real numbers is

(a) R (b) S (c) T (d) none of these

9. If A has 32 elements, B has 42 elements and $A \cup B$ has 62 elements, the number of elements in $A \cap B$ is

(a) 12 (b) 74 (c) 10 (d) none of these

10. In a group of 20 children, 8 drink tea but not coffee and 13 like tea. The number of children drinking coffee but not tea is

(a) 6 (b) 7 (c) 1 (d) none of these

11. The number of subsets of the sets $\{6, 8, 11\}$ is

(a) 9 (b) 6 (c) 8 (d) none of these

12. The sets $V = \{x / x+2=0\}$, $R=\{x / x^2+2x=0\}$ and $S = \{x : x^2 + x - 2 = 0\}$ are equal to one another if x is equal to

(a) -2 (b) 2 (c) ½ (d) none of these

13. If the universal set $E = \{x \mid x$ is a positive integer $<25\}$, $A = \{2, 6, 8, 14, 22\}$, $B = \{4, 8, 10, 14\}$ then

(a) $(A \cap B)' = A' \cup B'$ (b) $(A \cap B)' = A' \cap B'$ (c) $(A' \cap B)' = \phi$ (d) none of these

14. If the set P has 3 elements, Q four and R two then the set $P \times Q \times R$ contains

(a) 9 elements (b) 20 elements (c) 24 elements (d) none of these

15. Given $A = \{2, 3\}$, $B = \{4, 5\}$, $C = \{5, 6\}$ then $A \times (B \cap C)$ is

(a) $\{(2, 5), (3, 5)\}$ (b) $\{(5, 2), (5, 3)\}$ (c) $\{(2, 3), (5, 5)\}$ (d) none of these

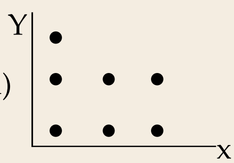
16. A town has a total population of 50,000. Out of it 28,000 read the newspaper X and 23,000 read Y while 4,000 read both the papers. The number of persons not reading X and Y both is

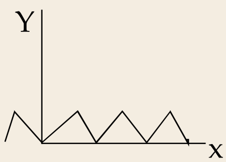
(a) 2,000 (b) 3,000 (c) 2,500 (d) none of these

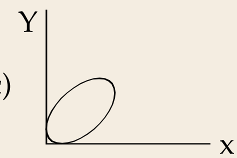
17. If $A = \{1, 2, 3, 5, 7\}$ and $B = \{1, 3, 6, 10, 15\}$. Cardinal number of A-B is

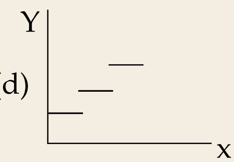
(a) 3 (b) 4 (c) 6 (d) none of these

18. Which of the diagram is graph of a function

(a) 

(b) 

(c) 

(d) 

19. At a certain conference of 100 people there are 29 Indian women and 23 Indian men. Out of these Indian people 4 are doctors and 24 are either men or doctors. There are no foreign doctors. The number of women doctors attending the conference is

(a) 2 (b) 4 (c) 1 (d) none of these

20. Let A = {a, b}. Set of subsets of A is called power set of A denoted by P(A). Now n(P(A) is

(a) 2 (b) 4 (c) 3 (d) none of these

21. Out of 2000 employees in an office 48% preferred Coffee (c), 54% liked (T), 64% used to smoke (S). Out of the total 28% used C and T, 32% used T and S and 30% preferred C and S, only 6% did none of these. The number having all the three is

(a) 360 (b) 300 (c) 380 (d) none of these

22. Referred to the data of Q. 21 the number of employees having T and S but not C is

(a) 200 (b) 280 (c) 300 (d) none of these

23. Referred to the data of Q. 21 the number of employees preferring only coffee is

(a) 100 (b) 260 (c) 160 (d) none of these

24. If f(x) = x + 3, g(x) = x^2, then g of (x) is

(a) $(x + 3)^2$ (b) $x^2 + 3$ (c) $x^2 (x + 3)$, (d) none of these

25. If f(x) = 1/1–x, then $f^{-1}(x)$ is

(a) 1–x (b) (x–1)/x (c) x/(x–1) (d) none of these

ANSWERS

Exercise 7(A)

1. b	**2.** a	**3.** c	**4.** a	**5.** b	**6.** c	**7.** b	**8.** c
9. a	**10.** b	**11.** b	**12.** a	**13.** c	**14.** b	**15.** b	**16.** a
17. b	**18.** c	**19.** b	**20.** c	**21.** a	**22.** b	**23.** c	**24.** b
25. b	**26.** a	**27.** a	**28.** b	**29.** c	**30.** a	**31.** b	**32.** a

Exercise 7(B)

1. b, d	**2.** c	**3.** a	**4.** b	**5.** a	**6.** c	**7.** b	**8.** b
9. a	**10.** b	**11.** d	**12.** a	**13.** b	**14.** a	**15.** a	**16.** b
17. a	**18.** c	**19.** b	**20.** a				

Exercise 7(C)

1. a	**2.** d	**3.** d	**4.** b	**5.** a	**6.** d	**7.** d	**8.** d
9. a	**10.** b	**11.** c	**12.** a	**13.** a	**14.** c	**15.** a	**16.** b
17. a	**18.** b	**19.** c	**20.** b	**21.** a	**22.** b	**23.** c	**24.** a
25. b							

7.7 CONCEPT OF LIMIT

I) We consider a function f(x) = 2 x. If x is a number approaching to the number 2 then f(x) is a number approaching to the value 2x2= 4

The following table shows f(x) for different values of x approaching 2

x	f(x)
1.90	3.8
1.99	3.98
1.999	3.998
1.9999	3.9998
2	4

Here x approaches 2 from values of x<2 and for x being very close to 2 f(x) is very close 4. This situation is defined as left-hand limit of f(x) as x approaches 2 and is written as lim f(x) = 4, $x \to 2-$

x	f(x)
2.0001	4.0002
2.001	4.002
2.01	1.02
2.0	4

Here x approaches 2 from values of x greater than 2 and for x being very close 2f(x) is very close to 4. This situation is defined as right-hand limit of f(x) as x approaches 2 and is written as lim f(x)= 4 as $x \to {}_{2+}$

So we write

$$\lim_{x \to 2-} f(x) = \lim_{x \to 2-+} f(x) = 4$$

Thus $\lim_{x \to a} f(x)$ is said to exist when both left-hand and right-hand limits exists and they are equal. We write as

$$\lim_{x \to a-} f(x) = \lim_{x \to a-+} f(x) = \lim_{x \to a} f(x)$$

Thus $\lim_{x \to a} f(x)$ is said to exist when both left-hand and right-hand limits exists and they are equal. We write as

$$\lim_{x \to a} f(x) = \lim_{x \to a+} f(x) = \lim_{x \to a} f(x)$$

Thus, if $\lim_{h \to o} f(a + h) = \lim_{h \to o} f(a - h)$, $\quad (h > o)$

then $\lim_{x \to a} f(x)$ exists

We now consider a function defined by

$$fx = \begin{cases} 2x-2 & \text{for } x < 0 \\ 1 & \text{for } x = 0 \\ 2x+2 & \text{for } x > 0 \end{cases}$$

We calculate limit of f(x) as x tend to zero. At x =0 , f(x) =1 (given). If x tends to zero from left-hand side for the value of x<0, f(x) is approaching (2x0) $-2 + -2$ which is defined as left-hand limit of f(x) as $x \to 0$. We can write this as

$$\lim_{x \to 0-} = -2$$

Similarly if x approaches zero from right-hand side for values of x>0, f(x) is approaching 2 x 0 + 2 = 2. We can write this as $\lim_{x \to 0+} f(x) = -2$.

In this case both left-hand and right-hand exist, but they are not equal. So we may conclude that $\lim_{x \to 0} f(x)$ does not exist.

7.8. USEFUL RULES (THEOREMS) ON LIMITS

Let $\lim_{x \to a} f(x) = l$ and $\lim_{x \to a} g(x) = m$

where l and m are finite quantities

i) $\lim_{x \to a} f\{f(x) + g(x)\} = \lim_{x \to a} f(x) + \lim_{x \to a} g(x) = l + m$

That is limit of the sum of the difference of two functions is equal to the sum of their limits

ii) $\lim_{x \to a} \{f(x) - g(x)\} = \lim_{x \to a} f(x) - \lim_{x \to a} g(x) = l - m$

That is limit of the difference of two functions is equal to difference of their limits.

iii) $\lim_{x \to a} \{f(x) .g(x)\} = \lim_{x \to a} f(x) .\lim_{x \to a} g(x) = lm$

That is limit of the product of two functions is equal to the product of their limits

iv) $\lim_{x \to a} \{f(x) / g(x)\} = \left\{\lim_{x \to a} f(x)\right\} / \left\{\lim_{x \to a} g(x)\right\} = l / m$ if $m \neq 0$

That is limit of the quotient of two functions is equal to the quotient of their limits.

v) $\lim_{x \to a} c = c$ where c is constant

That is limit of a constant is the constant

vi) $\lim_{x \to a} cf(x) = c \lim_{x \to a} f(x)$

vii) $\lim_{x \to a} F\{f(x)\} = F\{\lim_{x \to} F(x)\} = F(l)$

viii) $\lim_{x \to 0+} \frac{1}{x} = \lim_{h \to 0} \frac{1}{h} \to +\infty$ (h>0)

$\lim_{x \to 0+} \frac{1}{x} = \lim_{h \to 0} \frac{1}{-h} \to -\infty$ (h>0)

∞ is a very-every large number called infinity

Thus $\lim_{x \to 0+} \frac{1}{x}$ does not exist.

Example 1: Evaluate (i) $\lim_{x \to 2}(3x+9)$; (ii) $\lim_{x \to 5} \frac{1}{x-1}$ (iii) $\lim_{x \to a} \frac{1}{x-a}$

Solution:

(i) $\lim_{x \to 2} (3x+9) = 3.2+9 = (6+9) = 15$

(ii) $\lim_{x \to 5} \frac{1}{x-1} = \frac{1}{5-1} = \frac{1}{4}$

(iii) $\lim_{x \to a} \frac{1}{x-a}$ does not exist, since $\lim_{x \to a+} \frac{1}{x-a} \to +\infty$ and $\lim_{x \to a-} \frac{1}{x-a} \to -\infty$

Example-2 Evaluate $\lim_{x \to 2} \frac{x^2-5x+6}{x-2}$

Solution: At x = 2 the function becomes undefined as 2-2 =0 and division by zero is not mathematically defined.

So $\lim_{x \to 2}\left\{x^2-5x+6/(x-2)\right\} = \lim_{x \to 2}\{(x-3)/(x-2)\} = \lim_{x \to 2}(x-=3)$ $(\because x-2 \neq 0)$

= 2-3 = -1

Example-2 Evaluate $\lim_{x \to 2} \frac{x^2+2x-1}{\sqrt{x^2+2}}$

$$\lim_{x \to 2} \frac{x^2+2x-1}{\sqrt{x^2+2}} = \frac{\lim_{x \to 2}(x^2+2x-1)}{\lim_{x \to 2}\sqrt{x^2+2}} = \frac{\lim_{x \to 2}(x^2+\lim_{x \to 2} 2x-1)}{\sqrt{\lim_{x \to 2} x^2+2}}$$

$$\frac{(2)^2+2\text{x } 2-1}{\sqrt{(2)^2+2}} = \frac{7}{\sqrt{6}}$$

SOME IMPORTANT LIMITS

We now state some important limits

a) $\lim\limits_{x\to 0}\dfrac{(e^x -1)}{x}=1$

b) $\lim\limits_{x\to 0}\dfrac{a^x -1}{x}=\log_e \quad a(a>0)$

c) $\lim\limits_{x\to 0}\dfrac{\log(1+x)}{x}=1$

d) $\lim\limits_{x\to x}\left(1+\dfrac{1}{x}\right)^x=e$ or $\lim\limits_{x\to 0}\dfrac{(1+x)^{\frac{1}{x}}}{x}=e$

e) $\lim\limits_{x\to 0}\dfrac{x^n - a^n}{x}=na^{n-1}$

f) $\lim\limits_{x\to 0}\dfrac{(1+x)^n -1}{x}=n$

(A) The number e called exponential number is given by e = 2.718281828____________=2.7183. This number e is one of the useful constants in mathematics.

(B) In calculus all logarithms are taken with respect to base 'that is $\log\ x = \log_e\ x$.

ILLUSTRATIVE EXAMPLES

Example 1: Evaluate : $\lim\limits_{x\to 3}\dfrac{x^2-6x+9}{x-3}$, where $f(x)=\dfrac{x^2-6x+9}{x-3}$. Aso find f (3)

Solution: At x = 3 the function is undefined as division by zero is meaningless. While taking the limit as $x\to 3$, x cannot be exactly equal to 3 i.e. $x-3\neq 0$ and consequently division by x-3 is permissible.

Now $\lim\limits_{x\to 3}\dfrac{x^2-6x+9}{x-3}=\lim\limits_{x\to 3}\dfrac{(x-3)^2}{x-3}=\lim\limits_{x-3}(x-3)$ = 3-3 =0 $\quad f(3)\dfrac{0}{0}$ is undefined

The reader may compute the left-hand and the right-hand limits as an exercise.

Example 2: A function is defined as follows:

$$fx=\begin{cases}-3x \text{ when } x<0\\ 2x \text{ when } x>0\end{cases}$$

Test the existence of $\lim\limits_{x\to 0} f(x)$.

Solution: For x approaching 0 from the left $x < 0$.

Left-hand limit = $\lim_{x\to 0-} f(x) = \lim_{x\to 0-} (-3x) = 0$

When x approaches 0 from the right $x > 0$

Right-hand limit = $\lim_{x\to 0+} f(x) = \lim_{x\to 0+} 2x = 0$

Since L.H. limit = R.H. Limit, the limit exists. Thus, $\lim_{x\to 0} f(x) = 0$.

Example 3: $\lim_{x\to 3}\frac{x^2+4x+3}{x^2+6x+9}$.

Solution: $\frac{x^2+4x+3}{x^2+6x+9} = \frac{x^2+3x+x+3}{(x+3)^2} = \frac{x(x+3)+1(x+3)}{(x+3)^2} = \frac{(x+3)(x+1)}{(x+3)^2} = \frac{x+1}{x+3}$

$\therefore \lim_{x\to 3}\frac{x^2+4x+3}{x^2+6x+9} = \lim_{x\to 3}\frac{x+1}{x+3} = \frac{4}{3} = \frac{2}{3}$.

Example 4: Find the following limits:

(i) $\lim_{x\to 9}\frac{\sqrt{x}-3}{x-9}$ (ii) $\lim_{h\to 0}\frac{\sqrt{x+h}-\sqrt{x}}{h}$ if $h > 0$.

Solution:

(i) $\frac{\sqrt{x}-3}{x-9} = \frac{\sqrt{x}-3}{\left(\sqrt{x}+3\right)\left(\sqrt{x}-3\right)} = \frac{1}{\sqrt{x}+3}$. $\therefore \lim_{x\to 9}\frac{\sqrt{x}-3}{x-9} = \lim_{x\to 9}\frac{1}{\sqrt{x}+3} = \frac{1}{6}$.

(ii) $\frac{\sqrt{x+h}-\sqrt{x}}{h} = \frac{x+h-x}{h\left(\sqrt{x+h}+\sqrt{x}\right)} = \frac{1}{\sqrt{x+h}+\sqrt{x}} \therefore \lim_{h\to 0}\frac{\sqrt{x+h}-\sqrt{x}}{h} = \lim_{h\to 0}\frac{1}{\sqrt{x+h}+\sqrt{x}}$

$= \frac{1}{\lim_{h\to 0}\sqrt{x+h} + \lim_{h\to 0}\sqrt{x}} = \frac{1}{\sqrt{x}+\sqrt{x}} = \frac{1}{2\sqrt{x}.}$

Example 5: Find $\lim_{x\to 0}\frac{3x+|x|}{7x-5|x|}$.

Solution: Right-hand limit $= \lim_{x\to 0+}\frac{3x+|x|}{7x-5|x|} = \lim_{x\to 0+}\frac{3x+x}{7x-5x} = \lim_{x\to 0+} 2 = 2$

Left-hand limit $= \lim_{x\to 0-}\frac{3x+|x|}{7x-5|x|} = \lim_{x\to 0}\frac{3x-(x)}{7x-5(-x)} = \lim_{x\to 0-}\frac{1}{6} = \frac{1}{6}$.

Since Right-hand limit $\neq$ Left-hand limit the limit does not exist.

Example 6: Evaluate $=\lim\limits_{x\to 0}\frac{e^x - e^{-x}}{x}$

Solution: $=\lim\limits_{x\to 0}\frac{e^x - e^{-x}}{x} == \lim\limits_{x\to 0}\frac{(e^x-1)-(e^{-x}-1)}{x} = \lim\limits_{x\to 0}\frac{e^x - 1}{x} - = \lim\limits_{x\to 0}\frac{e^x - 1}{x} = 1-1=0$

Example 7: Find $\lim\limits_{x\to á}\left(1+\frac{9}{x}\right)^x$. (Form 1)

Solution: It may be noted that $\frac{x}{9}$ approaches α as x approaches ∞. i.e. $\lim\limits_{x\to\infty}\frac{x}{9}\to\infty$

$$\lim_{x\to\infty}\left(1+\frac{9}{x}\right)^x = \lim_{x/9\to\alpha}\left\{\left[\left(1+\frac{1}{\frac{x}{9}}\right)^{x/9}\right]^9\right\}$$

Substitution x / 9 = z the above expression takes the form $\lim\limits_{z\to\alpha}\left\{\left[\left(1+\frac{1}{z}\right)^z\right]^9\right\}$

$$=\left\{\lim_{z\to\infty}\left(1+\frac{1}{z}\right)^z\right\}^9 = e^9.$$

Example 8: Evaluate: $\lim\limits_{z\to\infty}\frac{2x+1}{x^3+1}$. $\left[\text{Form}\frac{\infty}{\infty}\right]$

Solution: As x approaches ∞, $2x+1$ and x^3+1 both approach ∞ and therefore the given function takes the form $\frac{\infty}{\infty}$ which is determinate. Therefore, instead of evaluating directly let us try for suitable algebraic transformation so that the indeterminate for is avoided.

$$\lim_{x\to\infty}\frac{x^{\frac{2}{2}+\frac{1}{3}}}{1+x^{\frac{1}{3}}} = \frac{\lim\limits_{x\to\infty}\left(x^{\frac{2}{2}+\frac{1}{3}}\right)}{\lim\limits_{x\to\infty}\left(1+x^{\frac{1}{3}}\right)} = \frac{\lim\limits_{x\to\infty}x^{\frac{2}{2}+}\lim\limits_{x\to\infty}x^{\frac{1}{3}}}{\lim\limits_{x\to\infty}1+\lim\limits_{x\to\infty}x^{\frac{1}{3}}} = \frac{0+0}{1+0} = \frac{0}{1} = 0$$

Example 9: Find $\lim\limits_{x\to\infty}\frac{1^2+2^2+3^2+.............+x^2}{x^3}$

$$\lim_{x\to\infty}\frac{[x(x+)(2x+1)]}{6x^3} = \frac{1}{6}\lim_{x\to\infty}\left\{\left(1+\frac{1}{x}\right)\left(2+\frac{1}{x}\right)\right\}$$

$$=\frac{1}{6}\text{ x } 1\text{ x }2 = \frac{1}{3}.$$

7.9 CONTINUITY:

By the term continuous we mean some thing which goes on without interruption and without abrupt changes. Here in mathematics the term continuous carries the same meaning. Thus, we define continuity of a function in the following way.

A function f(x) is said to be continuous at x= a if and only if

(i) f(x) is defined x= a

(ii) $\lim_{x \mapsto a-} f(x) = \lim_{x \mapsto a+} f(x)$

(iii) $\lim_{x \mapsto a} f(x) = f(a)$

In the second condition both left-hand and right-hand limits are equal. In the third condition limiting value of the function must be equal to its function value at x= a

Useful Information:

(i) The sum, difference and product of two continuous functions is a continuous function. This property holds good for any finite number of functions.

(ii) The quotient of two continuous functions is continuous function provided the denominator is not equal to zero.

Example:1 f(x) = [

$= \frac{3}{2} - x$ when $\frac{1}{2}$<x<1

$= \frac{1}{2}$ when x=$\frac{1}{2}$

Discuss the continuity of f(x) at x= $\frac{1}{2}$

$$\lim_{x \mapsto \frac{1}{2}-} f(x) = \lim_{x \to \frac{1}{2}+} \left(\frac{1}{2} - x\right) = \frac{1}{2} - \frac{1}{2} = 0$$

$$\lim_{x \mapsto \frac{1}{2}+} f(x) = \lim_{x \to \frac{1}{2}+} \left(\frac{3}{2} - x\right) = \frac{3}{2} - \frac{1}{2} = 1$$

Solution: Since LHL $\neq$ RHL

$\lim_{x \mapsto \frac{1}{2}} f(x)$ does not exist

Moreover f(1/2)

Hence f(x) is not continuous at x=1/2 , ie f(x) is discontinuous at x= 1/2

Example 2: Find points of discontinuity of the function f(x) = $\frac{x^2+2x+5}{x^2-3x+2}$

Solution: f(x) = $\frac{x^2+2x+5}{x^2-3x+2}$

For x=1 and x=2 the denominator becomes zero and the function f(x) is undefined at x=1 and x= 2. Hence the points of discontinuity are at x=1 and x= 2

Example 3: A function g(x) is defined as follows

g(x) = x when 0<x<1

=2-x when x≥1

is g(x) is continuous at x= 1

Solution:

$$\lim_{x\mapsto 1-} g(x) = \lim_{x\mapsto 1+} g(x)$$

$$\lim_{x\mapsto 1+} g(x) = \lim_{x\mapsto 1+}(2-x) = 2-1 = 1$$

$$\therefore \lim_{x\mapsto 1-} g(x) = \lim_{x\mapsto 1-} g(x) = 1$$

Also g(1) =2-1=1

and $\lim_{x\mapsto 1} g(x) = g(1) = 1$

Hence f(x) is coninuous at x=1

Example 4: The function f(x) = $\frac{x^2-9}{x-3}$ is undefined at x=3 . What value must be assigned to f (3), if f(x) is to be continuous at x=3

Solution: When x approaches 3, x≠3 ie x-3≠0

$$\lim_{x\mapsto 3} f(x) = \lim_{x\mapsto 3}\frac{(x-3)(x+3)}{(x-3)}$$

$$= \lim_{x\mapsto 3}(x+3) = 3+3 = 6$$

Therefore if f(x) is to be continuous at x=3 =f(3) = $\lim_{x\mapsto 3} f(x) = 6$

Example 5: Is the function f(x) = $|x|$ continuous at x= 0

Solution: We know that $|x|$ =x when x>0

= 0 when x= 0

= -x when x<0

$\lim_{x\to 0-} f(x) = \lim_{x\mapsto 0-}(-x) = 0$ and $\lim_{x\to 0+} f(x) = \lim_{x\mapsto 0+} x = 0$

Hence $\lim_{x\mapsto 0} f(x) = 0 = f(0)$

So f(x) is continuous at x = 0

ADDITIONAL QUESTION BANK

1. Following set notations represent: $A \subset B$; $x \notin A$; $A \supset B$; {0}; $A \not\subset B$

   (a) A is a proper subset of B; x is not an element of A; A contains B; singleton with an only element zero; A is not contained in B.

   (b) A is a proper subset of B; x is an element of A; A contains B; singleton with an only element zero; A is contained in B.

   (c) A is a proper subset of B; x is not an element of A; A does not contains B; contains elements other than zero; A is not contained in B.

   (d) None

2. Represent the following sets in set notation: – Set of all alphabets in English language, set of all odd integers less than 25, set of all odd integers, set of positive integers x satisfying the equation $x^2+5x+7=0$:

   (a) A = {x:x is an alphabet in English}, I = {x:x is an odd integer>25}, I = {2, 4, 6, 8 ….}
   I = $\{x: x^2+5x+7=0\}$

   (b) A={x:x is an alphabet in English}, I = {x:x is an odd integer<25}, I = {1, 3, 5, 7 ….}
   I = $\{x: x^2+5x+7=0\}$

   (c) A = {x:x is an alphabet in English}, I = {x:x is an odd integer £ 25}, I = {1, 3, 5, 7 ….}
   I = $\{x: x^2+5x+7=0\}$

   (d) None

3. Rewrite the following sets in a set builder form: - A={a, e, i, o, u} B={1, 2, 3, 4 ….} C is a set of integers between –15 and 15.

   (a) A={x:x is a consonant}, B={x:x is an irrational number}, C={x:–15<x<15∧ x is a fraction}

   (b) A={x:x is a vowel}, B={x:x is a natural number}, C={x: –15³x³15∧ x is a whole number}

   (c) A={x:x is a vowel}, B={x:x is a natural number}, C={x: –15 <x<15∧ x is a whole number}

   (d) None

4. If V={0, 1, 2, …9}, X={0, 2, 4, 6, 8}, Y={3, 5, 7} and Z={3 7} then
   $Y \cup Z, (V \cup Y) \cap X, (X \cup Z) \cup V$ are respectively: –

   (a) {3, 5, 7}, {0, 2, 4, 6, 8}, {0, 1, 2, …9} (b) {2, 4, 6}, {0, 2, 4, 6, 8}, {0, 1, 2, …9}

   (c) {2, 4, 6}, {0, 1, 2, …9}, {0, 2, 4, 6, 8} (d) None

5. In question No. (4) $(X \cup Y) \cap Z$ and $(\phi \cup V) \cap \phi$ are respectively: –

   (a) {0, 2, 4, 6, 8}, ϕ (b) {3, 7}, ϕ (c) {3, 5, 7}, ϕ (d) None

6. If V={x:x + 2 = 0} R={x:x^2+x–2=0, x <0} and S={x:x^2+4x+4=0} then V, R, S are equal for the value of x equal to ______.

   (a) 0 (b) –1 (c) –2 (d) None

7. What is the relationship between the following sets? A={x:x is a letter in the word *flower*} B={x:x is a letter in the word *flow*} C={x:x is a letter in the word *wolf*} D={x:x is a letter in the word *follow*}

   (a) B=C=D and all these are subsets of the set A

   (b) B=C≠D (c) B≠C≠D (d) None

8. Comment on the correctness or otherwise of the following statements: – (i) {a, b, c}={c, b, a} (ii) {a, c, a, d, c, d} ⊂ {a, c, d} (iii) {b} ∈ {{b}} (iv) {b} ⊂ {{b}} and ϕ ⊂ {{b}}.

   (a) Only (iv) is incorrect (b) (i) (ii) are incorrect

   (c) (ii) (iii) are incorrect (d) All are incorrect

9. If A={a, b, c}, B={a, b}, C = {a, b, d}, D={c, d} and E={d} state which of the following statements are correct: – (i) B ⊂ A (ii) D ≠ C (iii) C ⊃ E (iv) D ⊂ E (v) D ⊂ B (vi) D = A (vii) B ⊄ C (viii) E ⊂ A (ix) E ⊄ B (x) a ∈ A (xi) a ⊂ A (xii) {a} ∈ A (xiii) {a} ⊂ A

   (a) (i) (ii) (iii) (ix) (x) (xiii) only are correct (b) (ii) (iii) (iv) (x) (xii) (xiii) only are correct

   (c) (i) (ii) (iv) (ix) (xi) (xiii) only are correct(d) None

10. Let A = {0}, B = {0 1}, C = ϕ, D = {ϕ}, E = {x | x is a human being 300 years old}, F = {x | x ∈ A and x ∈ B} state which of the following statements are true: – (i) A ⊂ B (ii) B = F (iii) C ⊂ D (iv) C = E (v) A = F (vi) F = 1 and (vii) E = C = D

    (a) (i) (iii) (iv) and (v) only are true (b) (i) (ii) (iii) and (iv) are true

    (c) (i) (ii) (iii) and (vi) only are true (d) None

11. If A = {0, 1} state which of the following statements are true: – (i) {1} ⊂ A (ii) {1} ∈ A (iii) ϕ ∈ A (iv) 0 ∈ A (v) 1 ⊂ A (vi) {0} ∈ A (vii) ϕ ⊂ A

    (a) (i) (iv) and (vii) only are true (b) (i) (iv) and (vi) only are true

    (c) (ii) (iii) and (vi) only are true (d) None

12. State whether the following sets are finite, infinite or empty: – (i) X = {1, 2, 3, …..500} (ii) Y = {y: $y = a^2$; a is an integer} (iii) A = {x:x is a positive integer multiple of 2} (iv) B = {x:x is an integer which is a perfect root of 26<x<35}

    (a) finite infinite infinite empty (b) infinite infinite finite empty

    (c) infinite finite infinite empty (d) None

13. If A = {1, 2, 3, 4} B = {2, 3, 7, 9} and C = {1, 4, 7, 9} then

    (a) A ∩ B ≠ ϕ B ∩ C ≠ ϕ A ∩ C ≠ ϕ but A ∩ B ∩ C = ϕ

    (b) A ∩ B = ϕ B ∩ C = ϕ A ∩ C = ϕ A ∩ B ∩ C = ϕ

    (c) A ∩ B ≠ ϕ B ∩ C ≠ ϕ A ∩ C ≠ ϕ A ∩ B ∩ C ≠ ϕ

    (d) None

14. If the universal set is X = {x:x ∈ N, 1≤ x ≤ 12} and A = {1, 9, 10}, B = {3, 4, 6, 11, 12} and C = {2, 5, 6} are subsets of X then set A ∪ (B ∩ C) is _______.

(a) {3, 4, 6, 12} (b) {1, 6, 9, 10} (c) {2, 5, 6, 11} (d) None

15. As per question No.(14) the set (A ∪ B) ∩ (A ∪ C) is _________.

(a) {3, 4, 6, 12} (b) {1, 6, 9, 10} (c) {2, 5, 6, 11} (d) None

16. A sample of income group of 1172 families was surveyed and noticed that for income groups < ₹6000/-, 6000/- to ₹10999/-, ₹11000/-, to ₹15999/-, ₹16000 and above No. TV set is available to 70, 50, 20, 50 families one set is available to 152, 308, 114, 46 families and two or more sets are available to 10, 174, 84, 94 families.

If A = {x|x is a family owning two or more sets}, B = {x|x is a family with one set, }C = {x|x is a family with income less than ₹ 6000/-}, D = {x|x| is a family with income ₹ 6000/- to ₹ 10999/-}, E = {x|x is a family with income ₹ 11000/- to ₹ 15999/-}, find the number of families in each of the following sets (i) C ∩ B

(ii) A ∪ E

(a) 152, 580 (b) 152, 20 (c) 152, 50 (d) 152, 496

17. As per question No.(16) find the number of families in each of the following sets: –

(i) (A ∪ B)′ ∩ E (ii) (C ∪ D ∪ E) ∩ (A ∪ B)′

(a) 20, 50 (b) 152, 20 (c) 152, 50 (d) 20, 140

18. As per question No.(16) express the following sets in set notation: –

(i) {x|x is a family with one set and income of less than ₹ 11000/-}

(ii) {x|x is a family with no set and income over ₹ 16000/-}

(a) (C ∪ D) ∩ B (b) (A ∪ B)′ ∩ (C ∪ D ∪ E)′

(c) Both (d) None

19. As per question No.(16) express the following sets in set notation: –

(i) {x|x is a family with two or more sets or income of ₹ 11000/- to ₹ 15999/-}

(ii) {x|x is a family with no set}

(a) (A ∪ E) (b) (A ∪ B)′ (c) Both (d) None

20. If A = {a, b, c, d} list the element of power set P (a)

(a) {ϕ {a} {b}({c} {d} {a, b} {a, c} {a, d} {b, c} {b, d} {c, d}

(b) {a, b, c} {a, b, d} {a, c, d} {b, c, d}

(c) {a, b, c, d}

(d) All the above elements are in P (a)

21. If four members a, b, c, d of a decision making body are in a meeting to pass a resolution where rule of majority prevails list the wining coalitions. Given that a, b, c, d own 50%, 20%, 15%, 15% shares each.

(a) {a, b} {a, c} {a, d} {a, b, c} {a, b, d} {a, b, c, d} (b) {b, c, d}

(c) {b, c} {b, d} {c, d} {a, c, d} {b, c, d} {a} {b} {c} {d} ϕ (d) None

22. As per question No.(21) with same order of options (a) (b) (c) and (d) list the blocking conditions.

23. As per question No.(21) with same order of options (a) (b) (c) and (d) list the losing conditions.

24. If A ={a, b, c, d, e, f} B = {a, e, i, o, u} and C = {m, n, o, p, q, r, s, t, u} then $A \cup B$ is

(a) {a, b, c, d, e, f, i, o, u} (b) {a, b, c, i, o, u}

(c) {d, e, f, i, o, u} (d) None

25. As per question No.(24) $A \cup C$ is

(a) {a, b, c, d, e, f, m, n, o, p, q, r, s, t, u} (b) {a, b, c, s, t, u}

(c) {d, e, f, p, q, r} (d) None

26. As per question No.(24) $B \cup C$ is

(a) {a, e, i, o, u, m, n, p, q, r, s, t} (b) {a, e, i, r, s, t}

(c) {i, o, u, p, q, r} (d) None

27. As per question No.(24) A – B is

(a) {b, c, d, f} (b) {a, e, i, o} (c) {m, n, p, q} (d) None

28. As per question No.(24) $A \cap B$ is

(a) {a, e} (b) {i, o} (c) {o, u} (d) None

29. As per question No.(24) $B \cap C$ is

(a) {a, e} (b) {i, o} (c) {o, u} (d) None

30. As per question No.(24) $A \cup (B - C)$ is

(a) {a, b, c, d, e, f, i} (b) {a, b, c, d, e, f, o} (c) {a, b, c, d, e, f, u} (d) None

31. As per question No.(24) $A \cup B \cup C$ is

(a) {a, b, c, d, e, f, i, o, u, m, n, p, q, r, s, t} (b) {a, b, c, r, s, t}

(c) { d, e, f, n, p, q} (d) None

32. As per question No.(24) $A \cap B \cap C$ is

(a) ϕ (b) {a, e} (c) { m, n} (d) None

33. If A = {3, 4, 5, 6} B = {3, 7, 9, 5} and C = {6, 8, 10, 12, 7} then A′ is (given that the universal set U = {3, 4, ….., 11, 12, 13}

(a) {7, 8, ….12, 13} (b) {4, 6, 8, 10, ….13}

(c) { 3, 4, 5, 9, 11, 13} (d) None

34. As per question No.(33) with the same order of options (a), (b), (c) and (d) the set B′ is

35. As per question No.(33) with the same order of options (a), (b), (c) and (d) the set C′ is

36. As per question No.(33) the set (A′)′ is

(a) {3, 4, 5, 6} (b) {3, 7, 9, 5} (c) {8, 10, 11, 12, 13} (d) None

37. As per question No.(33) the set (B′)′ is

(a) {3, 4, 5, 6} (b) {3, 7, 9, 5} (c) {8, 10, 11, 12, 13} (d) None

38. As per question No.(33) the set $(A \cup B)'$ is

(a) {3, 4, 5, 6} (b) {3, 7, 9, 5} (c) {8, 10, 11, 12, 13} (d) None

39. As per question No.(33) the set $(A \cap B)'$ is

(a) {8, 10, 11, 12, 13} (b) {4, 6, 7, ….13} (c) {3, 4, 5, 7, 8,….13} (d) None

40. As per question No.(33) the set $A' \cup C'$ is

(a) {8, 10, 11, 12, 13} (b) {4, 6, 7, ….13} (c) {3, 4, 5, 7, 8,….13} (d) None

41. If A = {1, 2, …9}, B = {2, 4, 6, 8} C = {1, 3, 5, 7, 9}, D = {3, 4, 5} and E = {3, 5} what is set S if it is also given that $S \subset D$ and $S \not\subset B$

(a) {3, 5} (b) {2, 4} (c) {7, 9} (d) None

42 As per question No.(41) what is set S if it is also given that $S \subset B$ and $S \not\subset C$

(a) {3, 5} (b) {2, 4} (c) {7, 9} (d) None

43. If U = {1, 2, …9} be the universal set A = {1, 2, 3, 4} and B = {2, 4, 6, 8} then the set $A \cup B$ is

(a) {1, 2, 3, 4, 6, 8} (b) {2, 4} (c) {5, 6, 7, 8, 9} (d) {5, 7, 9}

44. As per question No.(43) with the same order of options (a) (b) (c) and (d) the set $A \cap B$ is

45. As per question No.(43) with the same order of options (a) (b) (c) and (d) the set A′ is

46. As per question No.(43) with the same order of options (a) (b) (c) and (d) the set $(A \cup B)'$ is

47. As per question No.(43) the set $(A \cap B)'$ is

(a) {1, 2, 3, 4, 6, 8} (b) {2, 4} (c) {5, 6, 7, 8, 9} (d) {1, 3, 5, 6, 7, 8, 9}

48. Let P = (1, 2, x), Q = (a x y), R = (x, y, z) then P × Q is

(a) {(1, a) (1, x) (1, y); (2, a) (2, x) (2, y); (x, a) (x, x) (x, y)}

(b) {(1, x); (1, y); (1, z); (2, x); (2, y); (2, z); (x, x) (x, y) (x, z)}

(c) {(a, x) (a, y) (a, z); (x, x) (x, y) (x, z); (y, x) (y, y) (y, z)}

(d) {(1, x) (1, y) (2, x) (2, y) (x, x) (x, y)}

49. As per question No.(48) with the same order of options (a) (b) (c) and (d) then the set P × R is

50. As per question No.(48) with the same order of options (a) (b) (c) and (d) then the set Q × R is

51. As per question No.(48) with the same order of options (a) (b) (c) and (d) then the set $(P \times Q) \cap (P \times R)$ is

52. As per question No.(48) the set $(R \times Q) \cap (R \times P)$ is

(a) {(a, x), (a, y), (a, z), (x, x), (x, y), (x, z), (y, x), (y, y), (y, z)}

(b) {(1, x), (1, y), (2, x), (2, y), (x, x), (x, y)}

(c) {(x, x), (y, x), (z, x)}

(d) {(1, a), (1, x), (1, y), (2, a), (2, x), (2, y), (x, a), (x, x), (x, y), (x, 1), (x, 2), (y, 1), (y, 2), (y, x), (z, 1), (z, 2), (z, x)}

53. As per question No. (48) with the same order of options (a) (b) (c) and (d) as in question No.(52) the set $(P \times Q) \cup (R \times P)$ is

54. If P has three elements Q four and R two how many elements does the Cartesian product set $P \times Q \times R$ will have

(a) 24 (b) 9 (c) 29 (d) None

55. Identify the elements of P if set Q = {1, 2, 3} and P × Q = {(4, 1), (4, 2), (4, 3), (5, 1), (5, 2), (5, 3), (6, 1), (6, 2), (6, 3)}

(a) {3, 4, 5} (b) {4, 5, 6} (c) {5, 6, 7} (d) None

56. If A = {2, 3}, B = {4, 5}, C = {5, 6} then $A \times (B \cup C)$ is

(a) {(2, 4), (2, 5), (2, 6), (3, 4), (3, 5), (3, 6)}

(b) {(2, 5), (3, 5)}

(c) {(2, 4), (2, 5), (3, 4), (3, 5), (4, 5), (4, 6), (5, 5), (5, 6)}

(d) None

57. As per question No.(56) with the same order of options (a) (b) (c) and (d) the set $A \times (B \cap C)$ is

58. As per question No.(56) with the same order of options (a) (b) (c) and (d) the set $(A \times B) \cup (B \times C)$ is

59. If A has 32 elements B has 42 elements and $A \cup B$ has 62 elements find the number of elements in $A \cap B$

(a) 74 (b) 62 (c) 12 (d) None

60. Out of a total population of 50,000 only 28,000 read Telegraph and 23,000 read Times of India while 4,000 read the both. How many do not read any paper?

(a) 3,000 (b) 2,000 (c) 4,000 (d) None

61. Out 2000 staff 48% preferred coffee 54% tea and 64% cocoa. Of the total 28% used coffee and tea 32% tea and cocoa and 30% coffee and cocoa. Only 6% did none of these. Find the number having all the three.

(a) 360 (b) 280 (c) 160 (d) None

62. As per question No.(61) with the same order of options (a), (b), (c) and (d) find the number having tea and cocoa but not coffee.

63. As per question No.(61) with the same order of options (a), (b), (c) and (d) find the number having only coffee.

64. Complaints about works canteen had been about Mess (M) Food (F) and Service (S). Total complaints 173 were received as follows: –

$n(M) = 110, n(F) = 55, n(S) = 67, n(M \cap F \cap S') = 20, n(M \cap S \cap F') = 11$

and $n(F \cap S \cap M') = 16$. Determine the complaints about all the three.

(a) 6 (b) 53 (c) 35 (d) None

65. As per question No. (64) with the same order of options (a), (b), (c) and (d) determine the complaints about two or more than two.

66. Out of total 150 students 45 passed in Accounts 50 in Maths. 30 in Costing 30 in both Accounts and Maths. 32 in both Maths and Costing 35 in both Accounts and Costing. 25 students passed in all the three subjects. Find the number who passed at least in any one of the subjects.

(a) 63 (b) 53 (c) 73 (d) None

67. After qualifying out of 400 professionals, 112 joined industry, 120 started practice and 160 joined as paid assistants. There were 32, who were in both practice and service 40 in both practice and assistantship and 20 in both industry and assistantship. There were 12 who did all the three. Find how many could not get any of these.

(a) 88 (b) 244 (c) 122 (d) None

68. As per question No. (67) with the same order of options (a) (b) (c) and (d) find how many of them did only one of these.

69. A marketing research team interviews 50 people about their drinking habits of tea coffee or milk or A B C respectively. Following data is obtained but the Manager is not sure whether these are consistent.

Category	**No.**	**Category**	**No.**
ABC	3	A	42
AB	7	B	17
BC	13	C	27
AC	18		

(a) Inconsistent since $42 + 17 + 27 - 7 - 13 - 18 + 3 \neq 50$

(b) Consistent

(c) Cannot determine due to data insufficiency

(d) None

70. On a survey of 100 boys it was found that 50 used white shirt 40 red and 30 blue. 20 were habituated in using both white and red shirts 15 both red and blue shirts and 10 blue and white shirts. Find the number of boys using all the colours.

(a) 20 (b) 25 (c) 30 (d) None

71. As per question No.(70) if 10 boys did not use any of the white red or blue colours and 20 boys used all the colours offer your comments.

(a) Inconsistent since no. of boys used all the three colours can't more then no. of boys used two colours

(b) Consistent

(c) Cannot determine due to data insufficiency

(d) None

72. Out of 60 students 25 failed in paper (1), 24 in paper (2), 32 in paper (3), 9 in paper (1) alone, 6 in paper (2) alone, 5 in papers (2) and (3) only and 3 in papers (1) and (2) only. Find how many failed in all the three papers.

(a) 10 (b) 60 (c) 50 (d) None

73. As per question No. (72) how many passed in all the three papers?

(a) 10 (b) 60 (c) 50 (d) None

74. Asked if you will cast your vote for a party the following feedback is obtained: –

	Yes	No	Don't know
Adult Male	10	20	5
Adult Female	20	15	5
Youth over 18 years	10	5	10

If A = set of Adult C = Common set of Women and Youth Y = set of positive opinion N = set of negative opinion then n(A′) is

(a) 25 (b) 40 (c) 20 (d) None

75. As per question No. (74) with the same order of options (a), (b), (c) and (d) the set $n(A \cap C)$ is

76. As per question No. (74) with the same order of options (a), (b), (c) and (d) the set $n(Y \cup N)'$ is

77. As per question No. (74) with the same order of options (a), (b), (c) and (d) the set $n[A \cap (Y \cap N)']$ is

78. In a market survey you have obtained the following data which you like to examine regarding its correctness:

Did not use the brand	April	May	June	April & May	May & June	April & June	April May June
Percentage answering 'Yes'	59	62	62	35	33	31	22

(a) Inconsistent since 59 + 62 + 62 – 35 – 33 – 31 + 22 ≠ 100

(b) Consistent

(c) cannot determine due to data insufficiency

(d) None

79. In his report an Inspector of an assembly line showed in respect of 100 units the following which you are require to examine.

Defect	Strength (S)	Flexibility (F)	Radius (R)	S and F	S and R	F and R	S F R
No. of pieces	35	40	18	7	11	12	3

(a) No. of pieces with radius defect alone was –2 which was impossible

(b) Report may be accepted

(c) Cannot be determined due to data insufficiency (d) None

80. A survey of 1000 customers revealed the following in respect of their buying habits of different grades:

A grade only	A and C grades	C grade	A grade but not B grade	A grade	C and B grades	None
180	80	480	230	360	80	140

How many buy B grade?

(a) 280 (b) 400 (c) 50 (d) None

81. As per question No. (80) with the same order of options (a) (b) (c) and (d) how many buy C grade if and only if they do not buy B grade?

82. As per question No. (80) with the same order of options (a) (b) (c) and (d) how many buy C and B grades but not the A grade?

83. Consider the following data: –

	Skilled & Direct Worker	Unskilled & Direct Worker	Skilled & Indirect Worker	Unskilled & Indirect Worker
Short Term	6	8	10	20
Medium Term	7	10	16	9
Long Term	3	2	8	0

If S M L T I denote short medium long terms skilled and indirect workers respectively find the number of workers in set M.

(a) 42 (b) 8 (c) 10 (d) 43

84. Consider the problem No. (83) and find the number of workers in set $L \cap I$.

(a) 42 (b) 8 (c) 10 (d) 43

85. Consider the problem No. (83) and find the number of workers in set $S \cap T \cap I$.

(a) 42 (b) 8 (c) 10 (d) 43

86. Consider the problem No. (83) and find the number of workers in set

$(M \cup L) \cap (T \cup I)$.

(a) 42 (b) 8 (c) 10 (d) 43

87. Consider the problem No. (83) and find the number of workers in set

$S' \cup (S' \cap I)'$.

(a) 42 (b) 44 (c) 43 (d) 99

88. Consider the problem No. (83). Find out which set of the pair has more workers as its members. Pair is $(S \cup M)'$ or L: –

(a) $(S \cup M)' > L$ (b) $(S \cup M)' < L$ (c) $(S \cup M)' = L$ (d) None

89. Consider the problem No. (88). Find out which set of the pair has more workers as its members. Pair is $(I \cap T)'$ or $S - (I \cap S')$: –

(a) $(I \cap T)' > [S - (I \cap S')]$ (b) $(I \cap T)' < [S - (I \cap S')]$

(c) $(I \cap T)' = [S - (I \cap S')]$ (d) None

90. Out of 1000 students 658 failed in the aggregate, 166 in the aggregate and in group-I 434 in aggregate and in group-II, 372 in group-I, 590 in group-II and 126 in both the groups. Find out how many failed in all the three.

(a) 106 (b) 224 (c) 206 (d) 464

91. As per question No.(90) how many failed in the aggregate but not in group-II?

(a) 106 (b) 224 (c) 206 (d) 464

92. As per question No.(90) how many failed in group-I but not in the aggregate?

(a) 106 (b) 224 (c) 206 (d) 464

93. As per question No.(90) how many failed in group-II but not in group-I?

(a) 106 (b) 224 (c) 206 (d) 464

94. As per question No.(90) how many failed in aggregate or group-II but not in group-I?

(a) 206 (b) 464 (c) 628 (d) 164

95. As per question No.(90) how many failed in aggregate but not in group-I and group-II?

(a) 206 (b) 464 (c) 628 (d) 164

ANSWERS

1.	(a)	**2.**	(b)	**3.**	(c)	**4.**	(a)	**5.**	(b)	**6.**	(c)
7.	(a)	**8.**	(a)	**9.**	(a)	**10.**	(a)	**11.**	(a)	**12.**	(a)
13.	(a)	**14.**	(b)	**15.**	(b)	**16.**	(d)	**17.**	(d)	**18.**	(c)
19.	(c)	**20.**	(d)	**21.**	(a)	**22.**	(b)	**23.**	(c)	**24.**	(a)
25.	(a)	**26.**	(a)	**27.**	(a)	**28.**	(a)	**29.**	(c)	**30.**	(a)
31.	(a)	**32.**	(a)	**33.**	(a)	**34.**	(b)	**35.**	(c)	**36.**	(a)
37.	(b)	**38.**	(c)	**39.**	(b)	**40.**	(c)	**41.**	(a)	**42.**	(b)
43.	(a)	**44.**	(b)	**45.**	(c)	**46.**	(d)	**47.**	(d)	**48.**	(a)
49.	(b)	**50.**	(c)	**51.**	(d)	**52.**	(c)	**53.**	(d)	**54.**	(a)
55.	(b)	**56.**	(a)	**57.**	(b)	**58.**	(c)	**59.**	(c)	**60.**	(a)
61.	(a)	**62.**	(b)	**63.**	(c)	**64.**	(a)	**65.**	(b)	**66.**	(b)
67.	(a)	**68.**	(b)	**69.**	(a)	**70.**	(b)	**71.**	(a)	**72.**	(a)
73.	(a)	**74.**	(a)	**75.**	(b)	**76.**	(c)	**77.**	(c)	**78.**	(a)
79.	(a)	**80.**	(a)	**81.**	(b)	**82.**	(c)	**83.**	(a)	**84.**	(b)
85.	(c)	**86.**	(d)	**87.**	(d)	**88.**	(c)	**89.**	(a)	**90.**	(a)
91.	(b)	**92.**	(c)	**93.**	(d)	**94.**	(c)	**95.**	(d)		